Electrons in Solids

Drude Model

Resistivity and Conductivity

\[ R = \frac{\rho L}{A} \]

\[ j = \frac{I}{A} \]

\[ V = E L \]

\[ E = j \rho \]

\[ j = \sigma E \]
Conductivity

Drude Model of Conductivity

\[ E = j\rho \]
\[ j = \sigma E \]
Drude Model of Conductivity

- Classical electrons behave as a thermal gas confined to a box.
- Outer valence electrons move freely about the metal.
- Collisions with other electrons are random and instantaneous.

Drude Model

- Interactions other than electron-electron and electron-ion collisions can be ignored.
- In the absence of an external field the average velocity of the electrons is zero.
- In the presence of an electric field the electrons are accelerated.
Drude Model

• Scattering with lattice ions creates an average “drag” force.

\[ m \frac{d\vec{v}}{dt} = e\vec{E} - \gamma m\vec{v} \]

Drude Model

• When these two forces cancel the electrons reach an equilibrium condition - they have reached a “terminal” or drift velocity, \( v_d \).

\[ m \frac{d\vec{v}}{dt} = e\vec{E} - \gamma m\vec{v}_d = 0 \]
Drude Model

- Rearrangement gives $\gamma$

$$\gamma = \frac{eE}{mv_d}$$

- $\mu$ is the electron mobility

$$\gamma = \frac{e}{m\mu}$$

Drude Model

- Solving for the velocity

$$v = v_d \left( 1 - e^{-t/\tau} \right)$$

$$\tau = \frac{mv_d}{eE} = \frac{1}{\gamma}$$
Drude

- $\tau$ is the “relaxation time” and is roughly the mean time between electron-ion collisions.

Drude Model

- If we have $n_f$ free-electrons then the current density is
  \[ j = n_f e v_d \]

- or in terms of the relaxation time
  \[ \tilde{j} = n_f e^2 \frac{\tau}{m} \tilde{E} \]
Drude Model

- Comparing with the definition of conductivity we find

\[ \sigma = \frac{n_f e^2 \tau}{m} \]

Electrons in Solids

Band Structure
Electrons Come Together

- Two electrons brought together from distant positions.
- Parity - mirror symmetry
- Energy differences between the two parity states.

Periodic Potential

- The potential felt by the electrons in a periodic lattice of atoms.
- States form “Bands” made up of many closely spaced energy levels (~E/N)
Band Structure

According to the Pauli Exclusion principle, two electrons cannot share the same state, so the electrons fill up the bands from the bottom.

The energy of the highest occupied level (at zero temperature) is called the Fermi Energy, $E_F$.

Fermi Energy

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- The energy of the highest occupied level (at zero temperature) is called the Fermi Energy, $E_F$. 
Non-Zero Temperature

- $T > 0$
- Electrons acquire energy and move above the Fermi level.

Conduction in Bands

- When an electric field is applied the potential energy at one end of the wire is raised in comparison to the other end.
- This means that for electrons in the uppermost states one side of the wire has unoccupied states that are lower in energy.
- The electrons move to that side - conduction.
An Electron in a Box

- Imagine an electron in a 1-D box.
- For a very large box the electrons will be approximately free inside the box.

\[ \psi = Ae^{-ikx} \]
\[ p = \hbar k \]
\[ E = \frac{\hbar^2 k^2}{2m} \]

Periodic Potential

- Kronig-Penney Model

\[ l = b + a = \text{lattice spacing} \]
Bloch Functions

• Wavefunction must also be periodic with the lattice.

\[ \psi = A u_k(x)e^{-ikx} \]

\[ u_k(x) = u_k(x + l) = u_k(x + nl) \]
Wave Number and Energy

- Dispersion curves give the relationship between energy and wave number (a measure of momentum).

Dispersion Curves
Insulator Dispersion Curve

Metal Dispersion Curve
Fermi Wave Vector

- The potential differs in each direction so in a 2-D or 3-D crystal we talk about wave vectors.
- The Fermi surface is defined by the wave vectors that have the same Fermi energy.

For metals

Temperatures Above Absolute Zero
“Ripples on the Fermi Sea”

- At $t > 0$ K electrons near the surface are promoted to states above the surface creating particle-hole pairs.
- Thermal energies are very small compared to the Fermi Energy.

Electrons in Solids

Defects and Phonons
Why resistivity?

- In Kronig-Penney we assumed perfect crystal hence no scattering.
- Real crystal
  - Impurities
  - Defects
  - Lattice Vibrations (phonons)

Phonons

- Lattice ions can move slightly about their position.
- Imagine a 1-D lattice:
  
- Any arrangement can be constructed out of a combination of “standing waves”
Phonons

- The vibrations of the lattice are quantized. There are a discrete set of frequencies that are allowed which can be used to build any possible lattice vibration.
- $E_{\text{phonon}} = hf$
- Increase in temperature creates energy that is used to create phonons.
- Electrons and phonons scatter from each other in the same way that photons and electrons scatter.
Electrons in Metals

Fermi Speed and Fermi Energy

Fermi Speed

- We can get a measure of the average speed of electrons in a metal by computing the Fermi speed.
- The conduction electrons have energies near the Fermi energy.
- Computations of Fermi energy in metals give a few eV which means the Fermi speed is on the order of $10^6$ m/s.
Fermi Speed and Conduction

- This Fermi speed implies a mean free path of about 100 lattice spacings (about right for defects, etc.)
- Electrons do not fall to zero speed in collisions because of Pauli exclusion principle.
- This explains why conductivity is independent of electric field (Ohm’s Law).

Thermal Speed

- Note the difference between Fermi speed and Thermal Speed.
- The classical relationship between temperature and speed for a point particle is

\[ \frac{1}{2} mv^2 = \frac{3}{2} k_B T \]

- The classical thermal speed is \( \sim 10^5 \) m/s