Section 7.1: Voting Systems

- **Plurality Method**

  The candidate who receives the greatest number of votes is the winner.

- **Borda Count Method**

  Each voter's last choice receives one point, each voter's second-to-last choice receives two points, and so on. The candidate with the most points is the winner.

- **Plurality with Elimination Method**

  We use a runoff election. In any given round, if a candidate wins a majority of the votes, they are the winner. Otherwise, the candidate (or candidates) with the fewest number of votes is (are) eliminated.

- **Pairwise Comparison Method**

  If Candidate X is preferred to Candidate Y by most voters, X gets one point. If they tie, they each get $\frac{1}{2}$ point. The candidate with the most points is the winner.

1. A club is preparing to elect its president. Three members are considering running for president: Alice, Bob, and Charles. Each of the five voters has a first, second, and third choice, as listed in the following table:

Voter:	Voter 1	Voter 2	Voter 3	Voter 4	Voter 5
1st choice:	Alice	Bob	Bob	Charles	Bob
2nd choice:	Charles	Alice	Charles	Alice	Alice
3rd choice:	Bob	Charles	Alice	Bob	Charles

Depending on who decides to run, the ballot will look different. For each of the following ballots, how many votes will each candidate receive? Who will win?

(a) Alice, Bob, and Charles are all on the ballot.

(b) Alice and Bob are on the ballot.

(c) Alice and Charles are on the ballot.

(d) Bob and Charles are on the ballot.

2. A company is electing a manager. Four candidates are running for the position: Micah, Darba, Jim, and Pam. The voters have the following preferences:

Ranking	Number of Ballots				
	3	2	4	3	2
1st choice	M	M	D	J	P
2nd choice	P	P	P	P	D
3rd choice	D	J	J	M	J
4th choice	J	D	M	D	M

(a) Here is the ballot:

Who should be the new manager?
☐ Micah
☐ Darba
☐ Jim
☐ Pam

Who will be elected manager? What percent of the vote will he or she have?

(b) As you look at the voters' preferences, who seems like the best choice to make the most voters happy?

(c) If your answers to parts (a) and (b) are different, can you explain why?

(d) Can you think of a different voting system that would help the voters elect your choice from part (b)?

3. If the company runs their election as a Borda count, who will win?

Ranking	Number of Ballots				
	3	2	4	3	2
1st choice	M	M	D	J	P
2nd choice	P	P	P	P	D
3rd choice	D	J	J	M	J
4th choice	J	D	M	D	M

4. A company warehouse is voting for its new warehouse manager, and the candidates are Dante, Creed, Phyllis, and Angela.

	# of Ballots			
Ranking	3	10	9	8
1st choice	D	C	A	P
2nd choice	P	P	D	A
3rd choice	C	D	P	D
4th choice	A	A	C	C

(a) Each member votes for his or her favorite candidate. What percentage of votes did the first-place candidate receive?

(b) Rather than elect a candidate who didn't receive a majority vote, the warehouse holds a run-off election between the top two candidates. Who will win the run-off election? Does this seem fair?

(c) Can you think of a better runoff system for the warehouse than a runoff between just the top two candidates? What result does it give?

5. The Army Corps of Engineers is trying to decide on a flood prevention project for the metro area of Margo-Foorhead. The options are

(D) A diversion on the North Dakota side
(R) Retention and reservoir system along the Red River valley
(F) Building floodwalls to 50′ throughout both cities
(C) Relocating the community to higher ground in, say, Colorado
(M) A diversion on the Minnesota side

The eighteen members of the Corps have the following preferences:

Ranking	Number of Ballots					
	5	4	2	3	1	3
1st choice	F	M	R	D	C	R
2nd choice	D	D	F	M	D	F
3rd choice	R	R	C	R	F	D
4th choice	M	C	D	F	M	M
5th choice	C	F	M	C	R	C

The ACE's voting rules employ *plurality with elimination.*

(a) What will the vote tallies be after the first round of voting? Is there a winner yet? If not, what should be eliminated?

(b) What will the vote tallies be after the second round of voting? Is there a winner yet? If not, what should be eliminated?

(c) What will the vote tallies be after the third round of voting? Is there a winner yet? If not, what should be eliminated?

6. If there are n candidates in an election conducted by plurality with elimination, what is the highest number of rounds of voting that could be required?

7. A company is voting on the Accountant-of-the-Month. The candidates are:

- Olivia
- Kevin
- Anja
- Toby

The preference table is the following:

	Number of Ballots				
Ranking	4	2	3	2	3
1st choice	A	T	K	O	O
2nd choice	O	O	A	K	A
3rd choice	K	K	T	A	T
4th choice	T	A	O	T	K

(a) Who would win in a head-to-head race between Olivia and Kevin?

(b) Who would win in a head-to-head race between Olivia and Anja?

(c) Fill in the following table of who would win head-to-head races; if there are any ties, put both names in the box.

	Toby	Anja	Kevin
Olivia			
Kevin			
Anja			

(d) Who won the most head-to-head races?
(Count any ties as $\frac{1}{2}$ a win for each contender.)

8. Our Math 105 class is voting on where to take a field trip. The options are:

   (N) National Security Agency, largest employer of mathematicians in the world

   (Z) Red River Zoo

   (W) West Acres

   (I) Ivers

   (K) Königsberg, for a walking tour of its bridges

   The preference table is the following.

Ranking	Number of Ballots					
	5	4	4	3	2	1
1st choice	N	K	K	Z	W	N
2nd choice	Z	N	Z	W	N	I
3rd choice	I	I	I	N	Z	K
4th choice	W	Z	N	K	K	W
5th choice	K	W	W	I	I	Z

   We decide to use the pairwise comparison method to decide. Where will we go?

9. If you use the pairwise comparison method with n candidates, how many head-to-head races are there?