MATHEMATICAL VIGNETTES

1. An ancient Tuscan, nearing death, called in all of his sons to explain how to divide his money. First, to his eldest son he said, "You may have one gold florin from the estate and a seventh of what remains after that." Next, to his second eldest son he said, "Take two gold florins from what remain of the estate, and a seventh of what remains after that." To the third son he said, "You may take three gold florins and a seventh of what is left." In this way the old man gave each son one gold florin more than the previous son, and a seventh of what remained, down to the last son, who got all that was left. After following their dying father's command, the sons found that they had shared their inheritance equally. How many sons were there, and how large was the estate?

2. An old Egyptian died, leaving his three sons 17 camels. Each camel was of slightly different value, so that it was given a ranking from 1 to 17. Now the first son was to receive 9 camels, the second son 6, and the third 2. How should the camels be allocated if each son is to receive the correct number of camels, but the average rank of the camels given each son is to be equal?

3. Phidias put a pair of rabbits in a place surrounded on all sides by a wall. Assuming adequate amounts of food, how many pairs of rabbits can be produced from that pair in a year if it is supposed that every month each pair begets a new pair, which from the second month on becomes productive?

4. On an archeological dig on the west bank of the Nile, Masri uncovered an ancient oil lamp. Upon being uncapped, the lamp emitted a jinni that had been bottled up since the time of the pharaohs. Granted three wishes, Masri first said, "I would like to find the Jewel of the Nile." The jinni agreed and instantly produced nine identical-looking stones in front of Masri. "Where is the Jewel of the Nile?" asked Masri. The jinni replied, "It is embedded in one of these stones. You said you wished to find it. You may take only one of the stones with you, so be careful how you choose! Eight of the stones weigh the same, but the stone containing the jewel weighs slightly more." Annoyed, Masri whispered under his breath, "I wish I had a balance scale." Ping! A balance scale instantly appeared. "That was wish number two!" triumphed the jinni, "and since it is based on one wish, the scale may be used only once." "What, only once?" asked Masri, "I wish I had another scale." Pong! Another balance scale appeared, which, just like the first, could only be used once. Using each of the two scales exactly once, is it possible for Masri to select the slightly heavier stone containing the Jewel of the Nile from among the nine stones? Explain why or why not.

5. The Pharaoh offered his minister the following test. There were two reed baskets covered in linen, each of which contained either a one hundred gold amulets or a deadly asp whose venom would kill instantly. The minister was to uncover one basket. The first bore a sign that read, "At least one of these baskets contains one hundred gold amulets." The second basket had a sign that read, "A poisonous asp that will kill you instantly is in the first basket." The minister was told that either both signs were true or both were false. Which basket did the minister uncover?

6. After a fierce naval battle was interrupted by storm, three men under Themistocles and three men under Xerxes happened to wash up on the shore of a deserted island. Reluctantly they decide to walk together to find food. After a time they came to a river that they wished to cross. None of the six could swim, but all could row. Fortunately, there was a small rowboat available for use. So small, however, that only two people could cross in it at one time.

   The Athenians admitted that they were not going to kill any of their enemy; the Persians, however, made it very clear that if at any time any group of Persians outnumbered any group of Greeks on either side of the river, the Persians would kill them. For this reason, the Persians did not fear being in a group outnumbered temporarily by the Greeks, but the three Greeks decided that at no time should they allow any group of

Greeks to be outnumbered by the Persians during the crossing. Is it possible for the Persians and the Greeks to all cross the river using only the one boat, so that at no time do the Persians outnumber the Greeks on either side of the river?

7. There was a scandal afoot in the Roman Tribune: someone had leaked out the secret plans of an upcoming military campaign. Tiberius Gracchus immediately drew up a list of the five most likely suspects. Once they were all assembled, Tiberius began the inquest. No matter how unscrupulous the five men before him were, however, Tiberius knew that three of them always told the truth. Knowing this, he asked all of them who had leaked the plans.

   Bibulus spoke up, "It was either Licinius or Crassus."

   Licinius, outraged, shouted enigmatically, "Neither Gaius nor I are guilty!"

   Crassus then said, "Well both of you are lying!"

   This provoked Marius to respond, "Actually, one of them is lying and the other is telling the truth."

   Then Gaius said, "No, Marius, that is not true."

   Who was the true leaker?

8. Julius Caesar, pontifex maximus, and his consort Cleopatra VII, queen of Egypt, were famous for their toga parties celebrating the ides of March, where there was much good-natured backstabbing, er back*slapping*; some even shook hands with other guests. At one such party, five couples were present (this included the host and hostess). Although it is not clear who shook hands with whom, we know that no one shook hands with themselves and no one shook hands with his or her companion. Given these facts, a guest might not shake anyone's hand or might shake as many as eight other people's hands. At midnight, Caesar gathered the guests and asked the nine other people how many hands each of them had shaken. Much to his surprise, each person gave a different answer. That is, someone did not shake any hands, someone else shook one hand, someone else shook two hands, someone else shook three hands, and so on, down to the last person, who shook eight hands. Given this outcome, determine the exact number of hands that Cleopatra shook.