Section 7.3 Weighted Voting Systems

1. __Weight__: The number of votes each voter has.

2. **Example:** 4 partners start a new business. P_1 owns 8 shares, P_2 owns 7 shares, P_3 owns 3 shares, and P_4 owns 2 shares. If

$$1 \text{ share} = 1 \text{ vote}$$

represent this weighted voting system.

3. __Motion__: A yes/no vote.

4. __Quota__: The minimum number of votes needed to pass a motion.

5. **Example:** If the quota for $[8, 7, 3, 2]$ is a simple majority, find the quota. What is the quota if a two-thirds majority is required?

6. Given a quota q for n voters with weights $w_1, w_2, \cdots, w_n$, we represent this by

$$[q \,|\, w_1, w_2, \cdots, w_n].$$

7. **Example:** Consider the weighted voting system

$$[20 \,|\, 7, 5, 4, 4, 2, 2, 2, 1, 1].$$

(a) How many voters are there?

(b) What is the quota?

(c) What is the weight for voter P_2?

(d) If the first 4 voters vote for a motion, does the motion pass?

(e) If P_1 and P_2 vote against a motion, does the motion pass?

8. <u>Coalition</u>: A nonempty set of voters in a weighted voting system. For n voters, the possible number of coalitions is $2^n - 1$.

9. Example: In a weighted voting system with 6 voters, how many possible coalitions are there?

10. Given $[21\,|\,10, 8, 5, 3, 2]$, answer the following questions.

    (a) How many voters are there?

    (b) What is the quota?

    (c) What is the total number of possible coalitions?

    (d) Is the coalition $\{P_2, P_3, P_4, P_5\}$ a winning coalition?

    (e) List all of the winning coalitions.

    (f) What do you notice about voter P_5? P_1? P_2?

11. <u>Veto Power</u>: A voter has veto power if that voter can prevent any motion from passing by voting no.

12. Is weight the same as power? Consider the weighted voting systems

$$[12\,|\,13, 7, 2] \quad \text{and} \quad [19\,|\,8, 7, 3, 2].$$

13. <u>Dummy</u>: A voter with no power.

14. <u>Dictator</u>: A voter whose power is greater than or equal to the quota.

15. Example: Consider $[12|9, 5, 4, 2]$. Are there any dummies or dictators?

16. <u>Critical Voter</u>: A voter in a coalition is a critical voter for that coalition if the coalition needs that voter to win.

17. Example: Consider $[10|6, 5, 4, 2]$. Complete the following table:

Winning Coalition	Critical Voter

18. <u>Banzhaf Power</u>: The Banzhaf power of a voter equals the number of winning coalitions in which the voter is critical.

19. <u>Total Banzhaf Power</u>: The sum of Banzhaf powers of all voters in a weighted voting system.

20. Banzhaf Power Index $= \dfrac{\text{voter's Banzhaf power}}{\text{total Banzhaf power}}$.

To Calculate Voters' Banzhaf Power Indices:

STEP 1: Locate all winning coalitions in the weighted voting system.

STEP 2: Determine which voters are critical in each of the winning coalitions.

STEP 3: Calculate each voter's Banzhaf power by adding up the number of times each voter is critical.

STEP 4: Calculate the total Banzhaf power for the weighted voting system by adding all voters' Banzhaf powers.

STEP 5: Determine each voter's Banzhaf power index by dividing his/her Banzhaf power by the total Banzhaf power.

21. Example: Consider $[19|11, 9, 8, 5]$. Determine the Banzhaf power index for each voter.

1. Consider the weighted voting system $[14, 9, 8, 5]$.

   (a) What is the smallest reasonable quota for this system?

   (b) What is the largest reasonable quota for this system?

2. Describe anything you find unusual or interesting about the following weighted voting systems.

   (a) $[20|10, 10, 9]$

   (b) $[7|4, 2, 1]$

   (c) $[51|50, 49, 1]$

3. Calculate the Banzhaf Power Index for each voter in the weighted voting system $[51|32, 22, 12]$.

4. In the weighted voting system $[q|30, 29, 16, 8, 3, 1]$, if a two-thirds majority of votes is needed, what is q?

5. In the weighted voting system $[q|6, 5, 4, 3, 2, 1]$, what is the largest value of q so that no voter has veto power?

6. In the weighted voting system $[q|8, 5, 4, 1]$, if every voter has veto power what is q?

7. In 1958, the Treaty of Rome established the European Economic Community (EEC) and instituted a weighted voting system for the EEC's governance. The members at that time were France, Germany, Italy, Belgium, the Netherlands, and Luxembourg. The three largest countries (France, Germany and Italy) were each given a vote with weight 4, Belgium and the Netherlands had votes of weight 2 and Luxembourg's vote had weight 1. The quota was 12.

   Describe anything you find unusual or interesting about this weighted voting system.