Section 1.4B Counting: Combinations

1. A 10-member board self selects a president, a vice president, and a treasurer. In how many ways can this be done? (Order matters)

$$_{10}P_3 = \frac{10!}{(10-3)!} = \frac{10!}{7!} = 10 \times 9 \times 8 = 720.$$

2. A 10-member board self selects a subcommittee of 3. In how many ways can this be done? (Order no longer matters).

$$\frac{_{10}P_3}{3!} = \frac{10!}{(10-3)!3!} = \frac{10!}{7!3!} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120.$$

3. Combinations:

   A combination is an unordered arrangement of objects. If we want to take r things out of n total things in any order, then there are

$$_nC_r = \frac{_nP_r}{r!} = \frac{n!}{(n-r)!r!}$$

   ways to do this. In example 2 above, there are

$$_{10}C_3 = \frac{10!}{(10-3)!3!} = \frac{10!}{7!3!} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120$$

   ways.

4. Evaluate $_9C_5$

5. Evaluate $_5C_3$

6. How many different 5-card poker hands can be dealt from a standard deck of 52 cards?

7. In how many different ways can the 9-member US Supreme Court reach a 6-3 decision?

8. A quiz contains 5 true/false questions.

   - In how many ways can the quiz be completed?
   - How many of the ways from (a) contain exactly 3 correct answers?
   - How many of the ways from (a) contain at least 3 correct answers?

9. **A committee of 13 has 7 women and 6 men. In how many ways can a subcommittee of 5 be formed if it consists of:**
   - **all women?**
   - **any 5 people?**
   - **exactly 2 men and 3 women?**
   - **at least 1 woman?**
   - **at least 3 women?**
10. **A telemarketer makes 15 phone calls in 1 hour. In how many ways can the outcomes of the calls be 3 sales, 8 no-sales, and 4 answering machines?**