1.2 IVPs, Existence & Uniqueness

1. Theorem 1.2.1:

2. Consider $\frac{dy}{dx} = f(y)$, where f and $\frac{\partial f}{\partial y}$ are continuous on $(-\infty, \infty)$.

   If $y_1 = 4$, $y_2 = 2$, $y_3 = 0$ are all constant solutions, what can be said about a solution y satisfying $y(0) = 1$?

3. Consider $\frac{dy}{dx} = y(y-2)(y-3)$. What happens to a solution y through the point $(0, -1)$? $(0, 1)$? $(0, 3)$?

4. Consider $\dfrac{dy}{dx} = y^{2/3}$. Show that $y_1 = \dfrac{x^3}{27}$ is a solution through $(0, 0)$. Find a constant solution y_2 to the differential equation that also goes through $(0, 0)$. Why does uniqueness fail here?

5. Consider $\dfrac{dy}{dx} = -y^2$. Verify that $y = \dfrac{1}{x + c}$ is a family of solutions.

   (a) Find a solution satisfying $y(0) = 1$, and its maximal interval of existence I.

   (b) Find a solution satisfying $y(0) = -1$, and its maximal interval of existence I.

   (c) Find a solution satisfying $y(0) = -1/2$, and its maximal interval of existence I.

   (d) If y is a solution with $-1 < y(0) < -1/2$, explain its long-term behavior.

   (e) If y is a solution with $y(0) > 0$, explain its long-term behavior.

   (f) If y is a solution with $y(0) = 0$, explain its long-term behavior. What is its maximal interval of existence I?

   (g) Are there any constant solutions to this differential equation?