

16.4 Parametrized Surfaces and Surface Integrals

1. Motivation:
 2. Parametrized surfaces \mathbf{S} given by $\Phi(u, v) = (x(u, v), y(u, v), z(u, v))$:
 3. Determine the surface $\Phi(u, v) = (u \cos v, u \sin v, u)$.
 4. Parametrize the surface $x^2 + y^2 + z^2 = 25$.
 5. Identify the surface $\Phi(u, v) = (R \cos u, R \sin u, v)$.
 6. Tangent Plane: For $\Phi(u, v) = (x(u, v), y(u, v), z(u, v))$, form the tangent vectors \mathbf{T}_u and \mathbf{T}_v .

7. Find the tangent plane to the parametric surface $\Phi(u, v) = (u, 2v^2, u^2 + v)$ at the point $(2, 2, 3)$.

8. Surface Area:

9. Surface Integral:

10. Find the surface area of the portion of the cone $\Phi(u, v) = (u, u \cos v, u \sin v)$ for $u \in [0, 2]$, $v \in [0, 2\pi]$.

11. Find the surface area of the paraboloid $z = x^2 + y^2$ under the plane $z = 9$.

12. Evaluate the surface integral $\iint_S xz \, dS$, where S is the part of the plane $x + y + z = 1$ in the first octant.

13. Set up but do not evaluate the iterated integral representing $\iint_S xyz \, dS$, where S is the portion of the surface $y^2 = x$ between the planes $z = 0$, $z = 4$, $y = 1$, $y = 2$.