

15.4B Triple Integrals in Cylindrical or Spherical Coordinates

- Triple integrals in cylindrical coordinates: For a region \mathcal{W} of the form

$$\theta_1 \leq \theta \leq \theta_2, \quad \alpha(\theta) \leq r \leq \beta(\theta), \quad z_1(r, \theta) \leq z \leq z_2(r, \theta),$$

the triple integral $\iiint_{\mathcal{W}} f(x, y, z) dV$ is given by

$$\int_{\theta_1}^{\theta_2} \int_{\alpha(\theta)}^{\beta(\theta)} \int_{z_1(r, \theta)}^{z_2(r, \theta)} f(r \cos \theta, r \sin \theta, z) r dz dr d\theta.$$

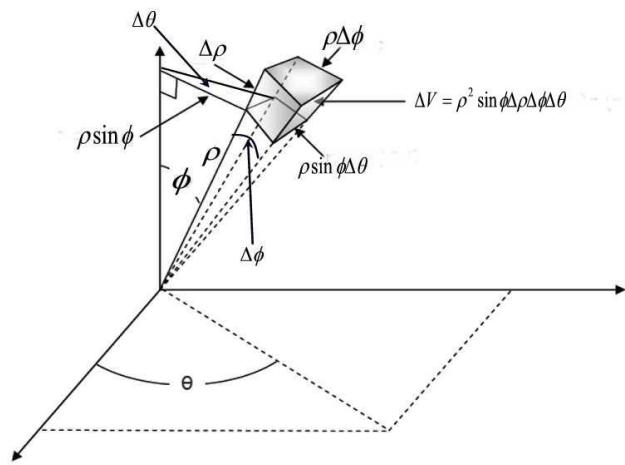
- Triple integrals in spherical coordinates: For a region \mathcal{W} of the form

$$\theta_1 \leq \theta \leq \theta_2, \quad \phi_1 \leq \phi \leq \phi_2, \quad \rho_1(\theta, \phi) \leq \rho \leq \rho_2(\theta, \phi),$$

the triple integral $\iiint_{\mathcal{W}} f(x, y, z) dV$ is given by

$$\int_{\theta_1}^{\theta_2} \int_{\phi_1}^{\phi_2} \int_{\rho_1(\theta, \phi)}^{\rho_2(\theta, \phi)} f(\rho \cos \theta \sin \phi, \rho \sin \theta \sin \phi, \rho \cos \phi) \rho^2 \sin \phi d\rho d\phi d\theta.$$

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1. Suppose an object is bounded above by the inverted paraboloid $z = 16 - x^2 - y^2$ and below by the xy -plane. If the density (mass per unit volume) of the object is given by $f(x, y, z) = 3\sqrt{x^2 + y^2}$, compute the total mass of the object using cylindrical coordinates.
2. Calculate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} (x^2 + y^2 + z^2)^2 dz dy dx$ using spherical coordinates.

3. Find the volume of the solid bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the hemisphere $z = 1 + \sqrt{1 - x^2 - y^2}$, using both cylindrical and spherical coordinates.

4. Find the volume of the solid outside the cone $x^2 + y^2 = z^2$ but inside the sphere $x^2 + y^2 + z^2 = 2$, using both cylindrical and spherical coordinates.