• Fubini's Theorem: If f(x,y,z) is continuous on the box  $\mathcal{B}$  defined by

$$\mathcal{B} = \{(x, y, z) : a \le x \le b, \ c \le y \le d, \ r \le z \le s\},\$$

then the triple integral of f over  $\mathcal{B}$  is

$$\iiint_{\mathcal{B}} f(x,y,z) \ dV = \int_{r}^{s} \int_{c}^{d} \int_{a}^{b} f(x,y,z) \ dx \ dy \ dz,$$

where the integrals are evaluated from the inside out.

• Theorem 2: Let  $\mathcal{D}$  be a region in the xy-plane. Assume that  $\psi(x,y)$  and  $\phi(x,y)$  are continuous with  $\psi(x,y) \leq \phi(x,y)$  for  $(x,y) \in \mathcal{D}$ . Then the triple integral of a continuous function f(x,y,z) over the domain

$$\mathcal{W} = \{(x, y, z) : (x, y) \in \mathcal{D}\}$$
 and  $\psi(x, y) \le z \le \phi(x, y)$ 

exists and is equal to the iterated integral

$$\iiint_{\mathcal{W}} f(x,y,z) \; dV = \iint_{\mathcal{D}} \left( \int_{z=\psi(x,y)}^{z=\phi(x,y)} f(x,y,z) \; dz 
ight) \; dA$$

1. Evaluate the triple integral  $\iiint_{\mathcal{B}} xyz^2 dV$ , where  $\mathcal{B}$  is the rectangular box

$$\mathcal{B} = \{(x, y, z): 0 \le x \le 1, -1 \le y \le 2, 0 \le z \le 3\}.$$

Integrate with respect to  $\boldsymbol{x}$ , then  $\boldsymbol{y}$ , then  $\boldsymbol{z}$ .

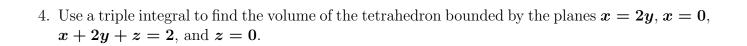
2. Evaluate  $\iiint_{\mathcal{W}} z \ dV$ , where  $\mathcal{W}$  is the solid tetrahedron bounded by the four planes x = 0, y = 0, z = 0, and x + y + z = 1.

3. Let  $\mathcal{W}$  be the region bounded by

$$z = 4 - y^2, \quad y = 2x, \quad z = 0, \quad x \ge 0.$$

Express  $\iiint_{\mathcal{W}} xyz \ dV$  as an iterated integral in three orders:

dz dx dy, dx dy dz, dy dx dz.



5. Find the volume of the solid in  $\mathbb{R}^3$  bounded by  $x = y^2$ ,  $y = x^2$ , z = x + y + 5, and z = 0.

6. Evaluate  $\iiint_{\mathcal{W}} \sqrt{x^2 + z^2} \ dV$ , where  $\mathcal{W}$  is the region bounded by the paraboloid  $y = x^2 + z^2$  and the plane y = 4.