1. Tangent Plane:

2. Tangent Plane and Normal Line:

3. Find the tangent plane and the normal line to the elliptic paraboloid  $z = 2x^2 + y^2$  at the point (1, 1, 3).

4. Find the tangent plane and the normal line to the surface  $z = \sqrt{4 - x^2 - 2y^2}$  at the point (1, -1, 1).

5. Find the tangent plane to the surface  $ze^z = x^2 - y^2$  at the point (1, 1, 0).

6. Linearization/Linear Approximations:

7. Find the linearization of the function  $f(x,y) = \sqrt{xy}$  at the point (4,16).

8. Find the linearization at (0,0) of the function  $f(x,y) = 1 + y + x \cos y$ .

9. If  $f(x,y) = x^2 + y^2$  and the local linearization to f at a point P is given by

$$L(x,y) = 2y - 2x - 2,$$

find  $\boldsymbol{P}$ .

10. Recall that for a function of one variable, y = f(x), if x changes from a to  $a + \Delta x$ , we defined the increment of y as

$$\Delta y = f(a + \Delta x) - f(a).$$

If  $\boldsymbol{f}$  is differentiable at  $\boldsymbol{a}$ , then

$$\Delta y = f'(a)\Delta x + \varepsilon \Delta x,$$

where  $\varepsilon \to 0$  as  $\Delta x \to 0$ .

11. Increment of z: If z = f(x, y) and x changes from (a, b) to  $(a + \Delta x, b + \Delta y)$ , then the increment of z is given by

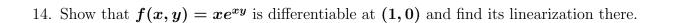
$$\Delta z = f(a + \Delta x, b + \Delta y) - f(a, b).$$

12. Differentiable: If z = f(x, y), then f is differentiable at (a, b) if  $\Delta z$  can be expressed in the form

$$\Delta z = f_x(a,b)\Delta x + f_y(a,b)\Delta y + \varepsilon_1\Delta x + \varepsilon_2\Delta y,$$

where  $\varepsilon_1$  and  $\varepsilon_2 \to 0$  as  $(\Delta x, \Delta y) \to (0, 0)$ .

13. Theorem: If the partial derivatives  $f_x$  and  $f_y$  exist near (a, b) and are continuous at (a, b), then f is differentiable at (a, b).



15. If 
$$f(x, y)$$
 is differentiable at  $(3, 4)$  with  $f(3, 4) = 5$ ,  $f_x(3, 4) = 2$  and  $f_y(3, 4) = -1$ , estimate the value of  $f(3.01, 3.98)$ .

16. Estimate the value of the function  $ze^z = x^2 - y^2$  at the point (1.1, 1.2).