14.4 Tangent Planes and Linear Approximations

1. Tangent Plane:

2. Tangent Plane and Normal Line:

3. Find the tangent plane and the normal line to the elliptic paraboloid $\boldsymbol{z = 2x^2 + y^2}$ at the point $\boldsymbol{(1, 1, 3)}$.

4. Find the tangent plane and the normal line to the surface $\boldsymbol{z = \sqrt{4 - x^2 - 2y^2}}$ at the point $\boldsymbol{(1, -1, 1)}$.

5. Find the tangent plane to the surface $\boldsymbol{ze^z = x^2 - y^2}$ at the point $\boldsymbol{(1, 1, 0)}$.

6. Linearization/Linear Approximations:

7. Find the linearization of the function $\boldsymbol{f(x, y) = \sqrt{xy}}$ at the point $\mathbf{(4, 16)}$.

8. Find the linearization at $\mathbf{(0, 0)}$ of the function $\boldsymbol{f(x, y) = 1 + y + x \cos y}$.

9. If $\boldsymbol{f(x, y) = x^2 + y^2}$ and the local linearization to $\boldsymbol{f}$ at a point $\boldsymbol{P}$ is given by

$$\boldsymbol{L(x, y) = 2y - 2x - 2},$$

find $\boldsymbol{P}$.

10. Recall that for a function of one variable, $y = f(x)$, if x changes from a to $a + \Delta x$, we defined the increment of y as

$$\Delta y = f(a + \Delta x) - f(a).$$

If f is differentiable at a, then

$$\Delta y = f'(a)\Delta x + \varepsilon \Delta x,$$

where $\varepsilon \to 0$ as $\Delta x \to 0$.

11. Increment of z: If $z = f(x, y)$ and x changes from (a, b) to $(a + \Delta x, b + \Delta y)$, then the increment of z is given by

$$\Delta z = f(a + \Delta x, b + \Delta y) - f(a, b).$$

12. Differentiable: If $z = f(x, y)$, then f is differentiable at (a, b) if Δz can be expressed in the form

$$\Delta z = f_x(a, b)\Delta x + f_y(a, b)\Delta y + \varepsilon_1 \Delta x + \varepsilon_2 \Delta y,$$

where ε_1 and $\varepsilon_2 \to 0$ as $(\Delta x, \Delta y) \to (0, 0)$.

13. Theorem: If the partial derivatives f_x and f_y exist near (a, b) and are continuous at (a, b), then f is differentiable at (a, b).

14. Show that $f(x, y) = xe^{xy}$ is differentiable at $(1, 0)$ and find its linearization there.

15. If $f(x, y)$ is differentiable at $(3, 4)$ with $f(3, 4) = 5$, $f_x(3, 4) = 2$ and $f_y(3, 4) = -1$, estimate the value of $f(3.01, 3.98)$.

16. Estimate the value of the function $ze^z = x^2 - y^2$ at the point $(1.1, 1.2)$.