14.1 Functions of Several Variables

1. A function of two variables is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$.

2. The set D is the domain of f and its range is the set of values that f takes on.

3. We also write $z = f(x, y)$. The variables x and y are independent variables and z is the dependent variable.

4. Example: Find the domain of the function $f(x, y) = \dfrac{2x + 3y}{x^2 + y^2 - 9}$.

5. Find the domain and range of $f(x, y) = \sqrt{4 - x^2 - y^2}$.

6. If f is a function of two variables with domain D, then the graph of f is the set of all points $(x, y, z) \in \mathbb{R}^3$ such that $z = f(x, y)$ and (x, y) is in D.

7. A linear function is a function $f(x, y) = ax + by + c$. The graph of a linear function is a plane.

8. Example: $f(x, y) = \sin x + \sin y$

9. Example: $f(x, y) = (x^2 + y^2)e^{-x^2 - y^2}$

10. Cobb-Douglas Production Function (1928): The total production (monetary value of all goods produced in one year) $\boldsymbol{P}$ is given by

$$\boldsymbol{P(L,K) = bL^{\alpha}K^{1-\alpha}},$$

where $\boldsymbol{L \geq 0}$ is the amount of labor and $\boldsymbol{K \geq 0}$ is the amount of capital, $\boldsymbol{\alpha}$ is a parameter between 0 and 1, and $\boldsymbol{b > 0}$ is the total factor productivity. For example, they used the formula

$$\boldsymbol{P(L,K) = 1.01\ L^{0.75}\ K^{0.25}}$$

to form a simple model of the American economy from 1899 – 1922.

11. Level Curves: The level curves of a function $\boldsymbol{f}$ of two variables are the curves with equations $\boldsymbol{f(x,y) = k}$ or simply $\boldsymbol{z = k}$, where $\boldsymbol{k}$ is constant.

12. $\boldsymbol{f(x,y) = \sin x + \sin y}$

13. $\boldsymbol{f(x,y) = (x^2 + y^2)e^{-x^2-y^2}}$

14. Find the level curves for $\boldsymbol{f(x,y) = x^2 + 2y^2}$.

15. Find the level curves for $\boldsymbol{f(x, y) = 6 - 3x - 2y}$.

16. Find the domain of $\boldsymbol{f(x, y) = \dfrac{3x + 10}{x^2 - y^2}}$.

17. Find the domain of $\boldsymbol{f(x, y) = \ln\left(\dfrac{1}{x} - y\right)}$.

18. Find the domain and range of $\boldsymbol{f(x, y) = \sin(x + y)}$.

19. Let $f(x, y) = x \ln(y^2 - x)$. Sketch

(a) the domain of f;

(b) $\{(x, y) : f(x, y) = 0\}$;

(c) $\{(x, y) : f(x, y) > 0\}$;

(d) $\{(x, y) : f(x, y) < 0\}$.