13.1 Vector-Valued Functions

1. Vector-Valued Function: A vector-valued function $\mathbf{r}(t)$ is a mapping from its domain $D \subset \mathbb{R}$ to its range $R \subset \mathbb{R}^3$ so that for each t in D we have $\mathbf{r}(t) = \mathbf{v}$ for exactly one vector $\mathbf{v} \in \mathbb{R}^3$. A vector-valued function can be written as

$$\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$$

using scalar functions f, g, h that are called component functions of $\mathbf{r}$.

2. Sketch the curve traced out by the endpoint of the two-dimensional vector-valued function

$$\mathbf{r}(t) = (t+1)\mathbf{i} + (t^2 - 2)\mathbf{j}.$$

3. Sketch the curve traced out by the endpoint of the vector-valued function

$$\mathbf{r}(t) = (4\cos t)\mathbf{i} - (3\sin t)\mathbf{j}, \quad t \in \mathbb{R}.$$

4. Sketch the curve traced out by the vector-valued function

$$\mathbf{r}(t) = (\sin t)\mathbf{i} - (3\cos t)\mathbf{j} + (2t)\mathbf{k}, \quad t \geq 0.$$

5. Sketch the curve traced out by the vector-valued function

$$\mathbf{r}(t) = \langle 3 + 2t, 5 - 3t, 2 - 4t \rangle, \quad t \in \mathbb{R}.$$

6. Match each of the vector-valued functions below with the corresponding computer-generated graph:

$$\mathbf{f}_1(t) = \langle \cos t, \ln t, \sin t \rangle \quad \mathbf{f}_2(t) = \langle t\cos t, t\sin t, t \rangle$$

$$\mathbf{f}_3(t) = \langle 3\sin 2t, t, t \rangle \qquad \mathbf{f}_4(t) = \langle 5\sin^3 t, 5\cos^3 t, t \rangle$$

7. Find parametric equations for the intersection of the cone $z = \sqrt{x^2 + y^2}$ and the plane $y + z = 2$.

8. Parametrize the circle of radius 2 with center $(1, 2, 5)$ that lies in a plane parallel to the $yz-$plane.

9. Parametrize the intersection of the plane $y = \frac{1}{2}$ with the sphere $x^2 + y^2 + z^2 = 1$.

10. Sketch the graph of $\mathbf{r}(t)$ and indicate the direction of increasing t for $\mathbf{r}(t) = (1+\cos t)\mathbf{i}+(3-\sin t)\mathbf{j}$, $0 \leq t \leq 2\pi$.

11. Describe the curve $\mathbf{r}(t) = (t \sin t)\mathbf{i} + (t \cos t)\mathbf{j} + t^2\mathbf{k}$ and its projections onto the $xy-$ and $yz-$planes. Find a circular paraboloid on which the curve lies.

12. For the helix $\mathbf{r}(t) = \langle a \cos t, a \sin t, ct \rangle$, find $c > 0$ so that the helix will make one complete turn in a distance of 3 units measured along the $z-$axis.

13. Show that the graph $\mathbf{r}(t) = \langle 3 \cos t, 3 \sin t, 3 \sin t \rangle$ lies on both a circular cylinder and a plane. What shape is it? Find the length of its major axis and its minor axis.