12.6 Survey of Quadric Surfaces

Quadric Surface: The graph of the equation

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + ax + by + cz + d = 0$$

in $\mathbb{R}^3$ (three-dimensional space), where $A, B, C, D, E, F, a, b, c, d$ are constants and at least one of A, B, C, D, E, F is not zero, is called a quadric surface.

1. Traces (Cross-Sections): Traces are curves of intersection of the surface with planes parallel to the coordinate planes.

2. Cylindrical Surface: Cylinders are surfaces whose traces in every plane parallel to a given plane are the same.

3. Sketch the surface $x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$.

4. Sketch the surface $x^2 + y^2 = z$.

5. Sketch the surface $x^2 + \frac{y^2}{4} = z^2$.

6. Sketch the surface $\frac{x^2}{4} + y^2 - \frac{z^2}{2} = 1$.

7. Sketch the surface $\frac{x^2}{4} - y^2 - \frac{z^2}{2} = 1$.

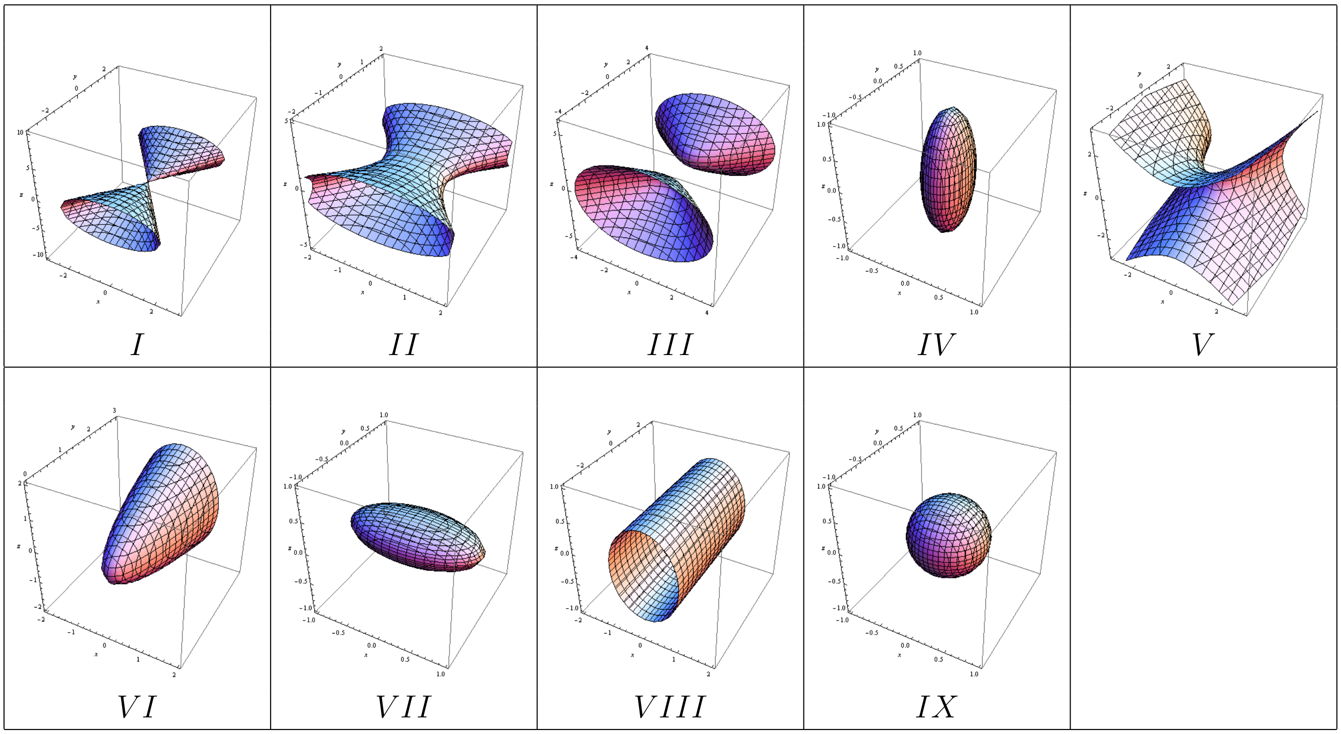
8. Sketch the surface $z = 2y^2 - x^2$.

9. Sketch the region bounded by the surfaces $z = \sqrt{x^2 + y^2}$ and $x^2 + y^2 = 1$ for $1 \leq z \leq 2$.

10. Sketch the region bounded by the surfaces $z = x^2 + y^2$ and $z = 2 - x^2 - y^2$.

11. Find an equation for the surface consisting of all points P for which the distance from P to the $x-$axis is twice the distance from P to the $yz-$plane. Identify the surface.

12. Match the graph with the appropriate equation below:



(a) $x^2 + 4y^2 + 9z^2 = 1$

(b) $9x^2 + 4y^2 + z^2 = 1$

(c) $3x^2 + 3y^2 + 3z^2 = 1$

(d) $x^2 - y^2 + z^2 = 1$

(e) $-x^2 + y^2 - z^2 = 1$

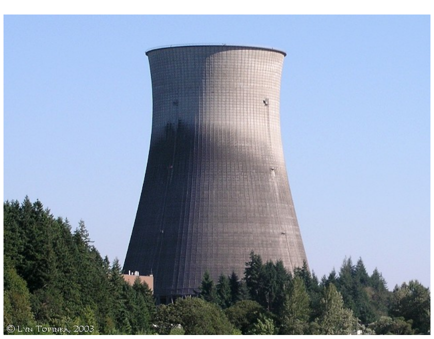
(f) $y = 2x^2 + z^2$

(g) $y^2 = 2x^2 + z^2$

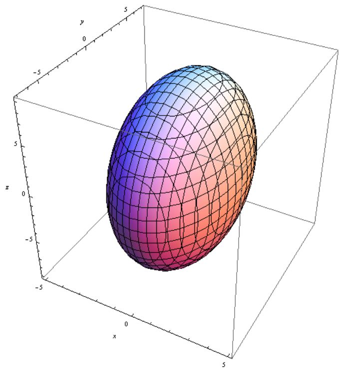
(h) $x^2 + 2z^2 = 1$

(i) $y = x^2 - z^2$

13. A cooling tower for a nuclear reactor is to be constructed in the shape of a hyperboloid of one sheet. The diameter at the base is 280 meters and the minimum diameter, 500 meters above the base, is 200 meters. Find an equation for the tower.

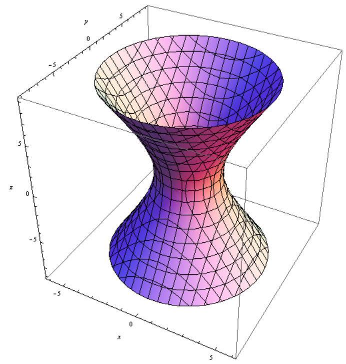


Ellipsoid



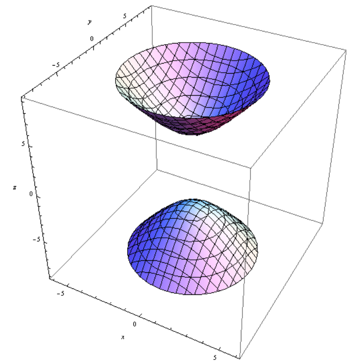
$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$$

Hyperboloid (one sheet)



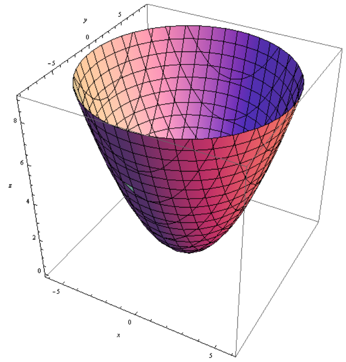
$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = \left(\frac{z}{c}\right)^2 + 1$$

Hyperboloid (two sheets)



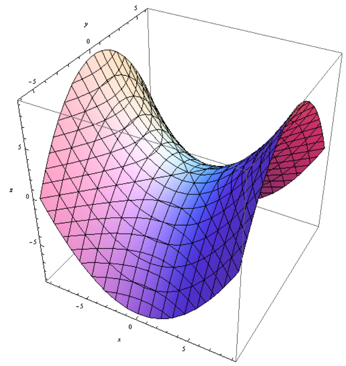
$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = \left(\frac{z}{c}\right)^2 - 1$$

Elliptic Paraboloid



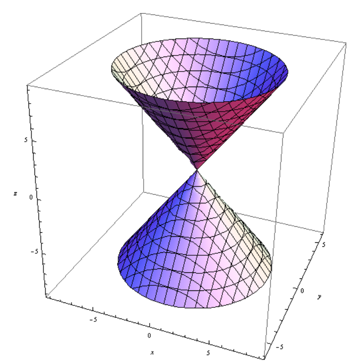
$$z = \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2$$

Hyperbolic Paraboloid



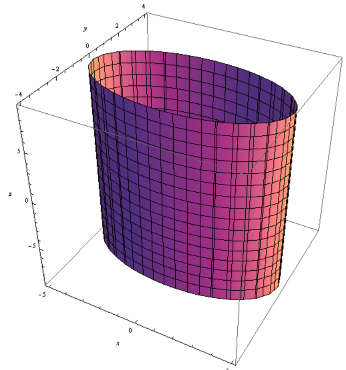
$$z = \left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2$$

Cone



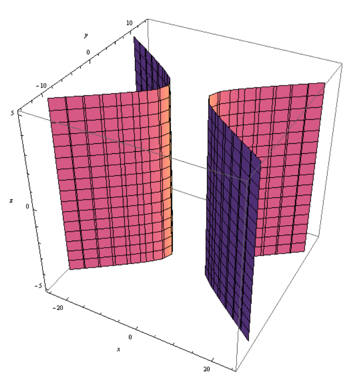
$$\left(\frac{z}{c}\right)^2 = \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2$$

Elliptic Cylinder



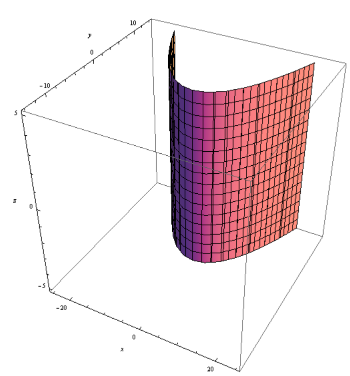
$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

Hyperbolic Cylinder



$$\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = 1$$

Parabolic Cylinder



$$y = ax^2$$