12.5 Planes and Spheres in Space

1.	Planes in \mathbb{R}^3 :	The vector	equation	of a	plane	with	direction	vectors	a and	b	through	the	point
	$P_0(x_0, y_0, z_0)$ is given by												

where
$$\mathbf{x} = \langle x, y, z \rangle$$
 and $\mathbf{p}_0 = \langle x_0, y_0, z_0 \rangle$.

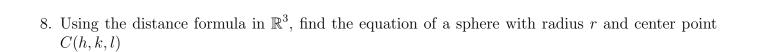
- 2. Definition of Normal: A vector \mathbf{n} is normal to a plane \mathscr{P} if \mathbf{n} is orthogonal to every vector lying in the plane, that is to say, $\mathbf{n} \cdot \mathbf{a} = 0$ for every vector \mathbf{a} lying in plane \mathscr{P} .
- 3. Normal Form of a Plane: If $\mathbf{n} = \langle a, b, c \rangle$ is orthogonal to a plane with direction vectors \mathbf{a} and \mathbf{b} through the point $P_0(x_0, y_0, z_0)$, then

where
$$\mathbf{x} = \langle x, y, z \rangle$$
 and $\mathbf{p}_0 = \langle x_0, y_0, z_0 \rangle$.

4. Find an equation of the plane through the points (-2,2,0), (-2,3,2), and (1,2,2).

- 5. Find an equation of the plane through the point (1,3,2) with normal vector (2,-1,5).
- 6. Find an equation of the plane through the point (4, -2, 3) and parallel to the plane 3x 7z = 12.

7. Two planes are parallel if their normal vectors are parallel.



9. Show that $x^2 + y^2 + z^2 - 6x + 4y - 2z = 11$ represents a sphere, and find its center and radius.

10. Find an equation of the sphere that passes through the point (4, 3, -1) and has center (3, 8, 1).

11. Find a sphere whose diameter has endpoints (-1,2,1) and (0,2,3).

12. Describe the set of all points whose coordinates satisfy

$$x^2 + y^2 + z^2 < 8 + 2x - 8z.$$