12.5 Planes and Spheres in Space

1. Planes in $\mathbb{R}^3$: The vector equation of a plane with direction vectors **a** and **b** through the point $P_0(x_0, y_0, z_0)$ is given by

   where $\mathbf{x} = \langle x, y, z\rangle$ and $\mathbf{p}_0 = \langle x_0, y_0, z_0\rangle$.

2. Definition of Normal: A vector **n** is normal to a plane $\mathscr{P}$ if **n** is orthogonal to every vector lying in the plane, that is to say, $\mathbf{n} \cdot \mathbf{a} = 0$ for every vector **a** lying in plane $\mathscr{P}$.

3. Normal Form of a Plane: If $\mathbf{n} = \langle a, b, c\rangle$ is orthogonal to a plane with direction vectors **a** and **b** through the point $P_0(x_0, y_0, z_0)$, then

   where $\mathbf{x} = \langle x, y, z\rangle$ and $\mathbf{p}_0 = \langle x_0, y_0, z_0\rangle$.

4. Find an equation of the plane through the points $(-2, 2, 0)$, $(-2, 3, 2)$, and $(1, 2, 2)$.

5. Find an equation of the plane through the point $(1, 3, 2)$ with normal vector $\langle 2, -1, 5\rangle$.

6. Find an equation of the plane through the point $(4, -2, 3)$ and parallel to the plane $3x - 7z = 12$.

7. Two planes are parallel if their normal vectors are parallel.

8. Using the distance formula in $\mathbb{R}^3$, find the equation of a sphere with radius r and center point $C(h, k, l)$

9. Show that $x^2 + y^2 + z^2 - 6x + 4y - 2z = 11$ represents a sphere, and find its center and radius.

10. Find an equation of the sphere that passes through the point $(4, 3, -1)$ and has center $(3, 8, 1)$.

11. Find a sphere whose diameter has endpoints $(-1, 2, 1)$ and $(0, 2, 3)$.

12. Describe the set of all points whose coordinates satisfy

$$x^2 + y^2 + z^2 < 8 + 2x - 8z.$$