12.2 Vectors in Space

1. We consider vectors in 3-dimensional Euclidean space $\mathbb{R}^3$, usually consisting of three rectangular coordinate axes x, y, z that intersect at the origin point $O = (0, 0, 0)$.

2. Plotting (a, b, c) for $a, b, c > 0$: Move a units along the x-axis from the origin O to the point $(a, 0, 0)$, then parallel to the y-axis b units to the point $(a, b, 0)$, then c units parallel to the z-axis to the point (a, b, c). Move similarly for other coordinates.

3. Plot the points $(1, -2, -3)$ and $(-3, 1, 2)$.

4. Distance: The distance between $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is given by

$$d\{P_1, P_2\} =$$

5. A position vector begins at the origin O and terminates at some point $A(a, b, c)$. Let $\mathbf{v} = \overrightarrow{OA} = \langle a, b, c\rangle$. We call a, b and c the components of $\mathbf{v}$. The magnitude or norm or length of $\mathbf{v}$ is

$$\|\mathbf{v}\| =$$

The vector with initial point $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is equivalent to the position vector

$$\overrightarrow{PQ} =$$

6. Vector addition and subtraction: For two position vectors $\mathbf{v} = \langle a_1, b_1, c_1\rangle$ and $\mathbf{w} = \langle a_2, b_2, c_2\rangle$ we have

$$\mathbf{v} + \mathbf{w} =$$

This is componentwise addition.

7. Scalar multiplication: For any scalar λ and position vector $\mathbf{v} = \langle a_1, b_1, c_1\rangle$, we have

$$\lambda\mathbf{v} =$$

Note that $\|\lambda\mathbf{v}\| =$.

8. Equality of two position vectors $\mathbf{v} = \langle a_1, b_1, c_1\rangle$ and $\mathbf{w} = \langle a_2, b_2, c_2\rangle$:

9. Parallel: Two vectors $\mathbf{v}$ and $\mathbf{w}$ are parallel if...

10. Standard basis vectors: For any position vector $\mathbf{v} = \langle a_1, a_2, a_3 \rangle$ we have

$$\mathbf{v} =$$

Define the standard basis vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ by

$$\mathbf{i} = \langle 1, 0, 0 \rangle, \quad \mathbf{j} = \langle 0, 1, 0 \rangle, \quad \text{and} \quad \mathbf{k} = \langle 0, 0, 1 \rangle.$$

Then $\mathbf{v} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$.

11. Unit vectors:

12. Sphere: A sphere is the set of all points at a constant distance (radius) from a given point (center). Therefore,

describes sphere with points (x, y, z) centered at (a, b, c) and radius r.

13. Parametric Equation of a Line in $\mathbb{R}^3$:

14. If ℓ is the line parametrized by $\mathbf{r}(t) = \langle 5 - 3t, -2 + t, 1 + 9t \rangle$, find the equation of the line through $P(-6, 4, -3)$ parallel to ℓ.

15. Find the distance between $(-1, 0, 2)$ and $(1, 2, 3)$.

16. Compute $\mathbf{v} + \mathbf{w}$, $2\mathbf{v} - 3\mathbf{w}$, and $\|3\mathbf{v} + \mathbf{w}\|$ for $\mathbf{v} = \langle -1, 0, 2 \rangle$ and $\mathbf{w} = \langle 1, 2, 3 \rangle$.

17. Find a unit vector in the direction of $2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$.

18. Find a vector with magnitude 6 in the direction of $\langle 2, 2, -1 \rangle$.

19. Identify the geometric shape described by the equation

$$x^2 + 2x + y^2 + z^2 - 4z = 0.$$

20. The thrust of an airplane's engine produces a speed of 966 kph in still air. The plane points in the direction of $\langle 2, 2, 1 \rangle$, and the wind is given by $\langle 16, -32, 0 \rangle$ kph. Find the velocity vector of the plane with respect to the ground and find the speed.

21. Are the points $P(2, 3, 1)$, $Q(4, 2, 2)$, and $R(8, 0, 4)$ on the same line?

22. Find a parametrization of the line through $P(3, 1, -2)$ and $Q(-2, 7, -4)$.

23. Do the lines $\ell_1 = \langle -2, 5, 1\rangle + t\langle 3, -4, 2\rangle$ and $\ell_2 = \langle 1, 3, -4\rangle + t\langle -1, 2, -3\rangle$ intersect?

24. Describe the surface $x + y = 2$.

25. Show that the triangle with vertices $P(-2, 4, 0)$, $Q(1, 2, -1)$, $R(-1, 1, 2)$ is equilateral.

26. Find the distance from $(3, 7, -5)$ to

    (a) $xy-$plane
    (b) $yz-$plane
    (c) $xz-$plane
    (d) $x-$axis
    (e) $y-$axis
    (f) $z-$axis

27. For a sphere centered at $(-1, 1, 2)$ with radius 3, find the point on the sphere

    (a) closest to the origin
    (b) farthest from the origin