

Homework 4.5 Curve Sketching (20 pts) Name_____.

1. Sketch the graph of a twice-differentiable function $y = f(x)$ with the following properties. Label coordinates on the graph where possible.

x	y	Derivatives
$x < 2$		$y' < 0, y'' > 0$
2	1	$y' = 0, y'' > 0$
$2 < x < 4$		$y' > 0, y'' > 0$
4	4	$y' > 0, y'' = 0$
$4 < x < 6$		$y' > 0, y'' < 0$
6	7	$y' = 0, y'' < 0$
$6 < x$		$y' < 0, y'' < 0$

2. Using calculus, find vertical/horizontal asymptotes, local extrema, and intervals of concavity for $f(x) = \frac{1}{x^2} + \frac{1}{x^3}$. Sketch the graph; be sure to label your graph and indicate the transition points (critical points and inflection points, if any).

3. Using calculus, find intervals where f is increasing/decreasing, where f is concave up/down, any inflection points, and any local maxima/minima. Then sketch a labeled graph of $y = f(x)$, indicating any maxima/minima and any inflection points on the graph:

$$f(x) = x^2 e^{-x/5}$$

4. Find the vertical asymptote, local extrema, and intervals of concavity for $f(x) = x + \frac{a}{x}$, where $a > 0$. Sketch the graph for generic $a > 0$; be sure to label your graph and indicate the transition points (critical points and inflection points, if any).