## 4.5 Curve Sketching

1. Using calculus, find intervals where $f$ is increasing/decreasing, where $f$ is concave up/down, any inflection points, and any local maxima/minima. Then sketch a labeled graph of $y = f(x)$, indicating any maxima/minima and any inflection points on the graph:

$$f(x) = (x^2 + 1)e^{-x/2}$$

2. Sketch the graph of a twice-differentiable function $y = f(x)$ with the following properties. Label coordinates on the graph where possible.

| $\boldsymbol{x}$ | $\boldsymbol{y}$ | **Derivatives** |
|---|---|---|
| $x < 2$ | | $y' < 0,\ y'' > 0$ |
| 2 | 1 | $y' = 0,\ y'' > 0$ |
| $2 < x < 4$ | | $y' > 0,\ y'' > 0$ |
| 4 | 4 | $y' > 0,\ y'' = 0$ |
| $4 < x < 6$ | | $y' > 0,\ y'' < 0$ |
| 6 | 7 | $y' = 0,\ y'' < 0$ |
| $6 < x$ | | $y' < 0,\ y'' < 0$ |

3. Find vertical/horizontal asymptotes, local extrema, and intervals of concavity for $f(x) = \dfrac{1}{x^2} + \dfrac{1}{x^3}$. Sketch the graph; be sure to label your graph and indicate the transition points (critical points and inflection points, if any).

4. Find the vertical asymptote, local extrema, and intervals of concavity for $f(x) = x + \frac{a}{x}$, where $a > 0$. Sketch the graph for generic $a > 0$; be sure to label your graph and indicate the transition points (critical points and inflection points, if any).