Section 1.6 Properties of Graphs

Math 110 Pre-Calculus

Concordia College, Moorhead, Minnesota

11 February 2019

Transformations (Review)

Vertical Shifts:

Transformations (Review)

Vertical Shifts: $\begin{cases} f(x) + c & : \\ & \end{cases}$

Transformations (Review)

Vertical Shifts: $\begin{cases} f(x) + c & : \text{shift up } c \text{ units} \\ & \end{cases}$

Transformations (Review)

Vertical Shifts: $\begin{cases} f(x)+c & \text{: shift up } c \text{ units} \\ f(x)-c & : \end{cases}$

Transformations (Review)

Vertical Shifts: $\begin{cases} f(x) + c & : \text{shift up } c \text{ units} \\ f(x) - c & : \text{shift down } c \text{ units} \end{cases}$

Transformations (Review)

Vertical Shifts: $\begin{cases} f(x)+c & \text{: shift up } c \text{ units} \\ f(x)-c & \text{: shift down } c \text{ units} \end{cases}$

Horizontal Shifts:

Transformations (Review)

Vertical Shifts: $\begin{cases} f(x)+c & : \text{shift up } c \text{ units} \\ f(x)-c & : \text{shift down } c \text{ units} \end{cases}$

Horizontal Shifts: $\begin{cases} f(x+c) & : \\ & \end{cases}$

Transformations (Review)

Vertical Shifts: $\begin{cases} f(x) + c & \text{: shift up } c \text{ units} \\ f(x) - c & \text{: shift down } c \text{ units} \end{cases}$

Horizontal Shifts: $\begin{cases} f(x + c) & \text{: shift left } c \text{ units} \\ & \end{cases}$

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$$\text{Vertical Shifts: } \begin{cases} f(x) + c & : \text{shift up } c \text{ units} \\ f(x) - c & : \text{shift down } c \text{ units} \end{cases}$$

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Vertical Shifts: $\begin{cases} f(x)+c & \text{: shift up } c \text{ units} \\ f(x)-c & \text{: shift down } c \text{ units} \end{cases}$

Horizontal Shifts: $\begin{cases} f(x+c) & \text{: shift left } c \text{ units} \\ f(x-c) & \text{: shift right } c \text{ units} \end{cases}$

Reflections:

Transformations (Review)

Vertical Shifts: $\begin{cases} f(x) + c & : \text{shift up } c \text{ units} \\ f(x) - c & : \text{shift down } c \text{ units} \end{cases}$

Horizontal Shifts: $\begin{cases} f(x + c) & : \text{shift left } c \text{ units} \\ f(x - c) & : \text{shift right } c \text{ units} \end{cases}$

Reflections: $\begin{cases} -f(x) & : \\ & \end{cases}$

Transformations (Review)

Vertical Shifts: $\begin{cases} f(x)+c & \text{: shift up } c \text{ units} \\ f(x)-c & \text{: shift down } c \text{ units} \end{cases}$

Horizontal Shifts: $\begin{cases} f(x+c) & \text{: shift left } c \text{ units} \\ f(x-c) & \text{: shift right } c \text{ units} \end{cases}$

Reflections: $\begin{cases} -f(x) & \text{: reflect across the } x\text{-axis} \\ & \end{cases}$

Transformations (Review)

$$\text{Vertical Shifts: } \begin{cases} f(x)+c & : \text{shift up } c \text{ units} \\ f(x)-c & : \text{shift down } c \text{ units} \end{cases}$$

$$\text{Horizontal Shifts: } \begin{cases} f(x+c) & : \text{shift left } c \text{ units} \\ f(x-c) & : \text{shift right } c \text{ units} \end{cases}$$

$$\text{Reflections: } \begin{cases} -f(x) & : \text{reflect across the } x\text{-axis} \\ f(-x) & : \end{cases}$$

Transformations (Review)

Vertical Shifts: $\begin{cases} f(x) + c & : \text{shift up } c \text{ units} \\ f(x) - c & : \text{shift down } c \text{ units} \end{cases}$

Horizontal Shifts: $\begin{cases} f(x + c) & : \text{shift left } c \text{ units} \\ f(x - c) & : \text{shift right } c \text{ units} \end{cases}$

Reflections: $\begin{cases} -f(x) & : \text{reflect across the } x\text{-axis} \\ f(-x) & : \text{reflect across the } y\text{-axis} \end{cases}$

Transformations (New)

Vertical Stretch/Compression:

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$$\begin{cases} cf(x), c > 1 \quad\quad : \\ \\ \\ \end{cases}$$

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Vertical Stretch/Compression:

$$\begin{cases} cf(x), c > 1 & \text{: stretch vertically by a factor of } c \text{ units} \\ & \quad \text{away from the } x\text{-axis} \\ & \\ & \end{cases}$$

Transformations (New)

Vertical Stretch/Compression:

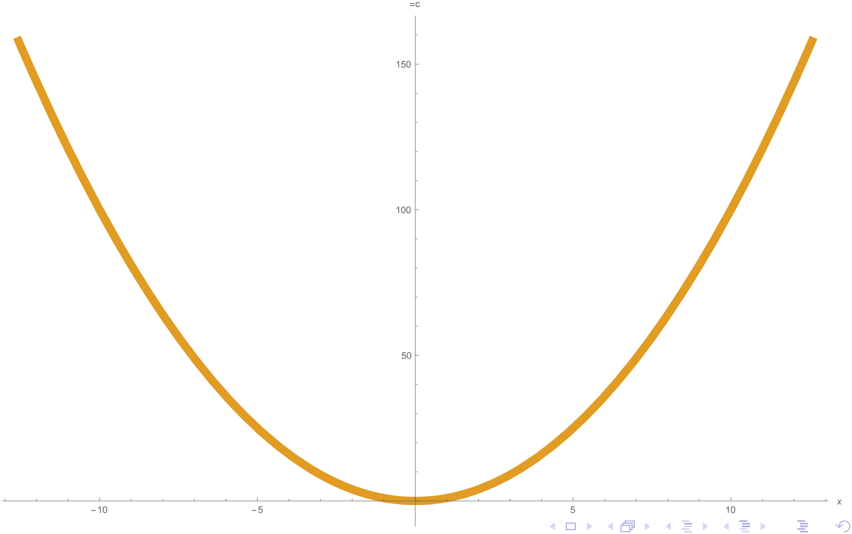
$$\begin{cases} cf(x), c > 1 & : \text{stretch vertically by a factor of } c \text{ units} \\ & \text{ away from the } x\text{-axis} \\ \\ cf(x), 0 < c < 1 & : \\ \\ \end{cases}$$

Transformations (New)

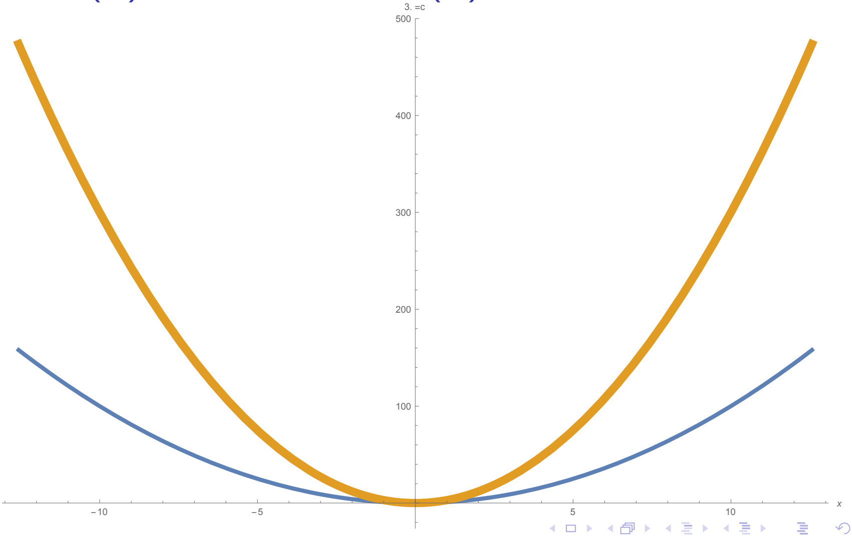
Vertical Stretch/Compression:

$$\begin{cases} cf(x), c > 1 & : \text{stretch vertically by a factor of } c \text{ units} \\ & \quad \text{away from the } x\text{-axis} \\ \\ cf(x), 0 < c < 1 & : \text{compress vertically by a factor of } c \text{ units} \\ & \quad \text{toward the } x\text{-axis} \end{cases}$$

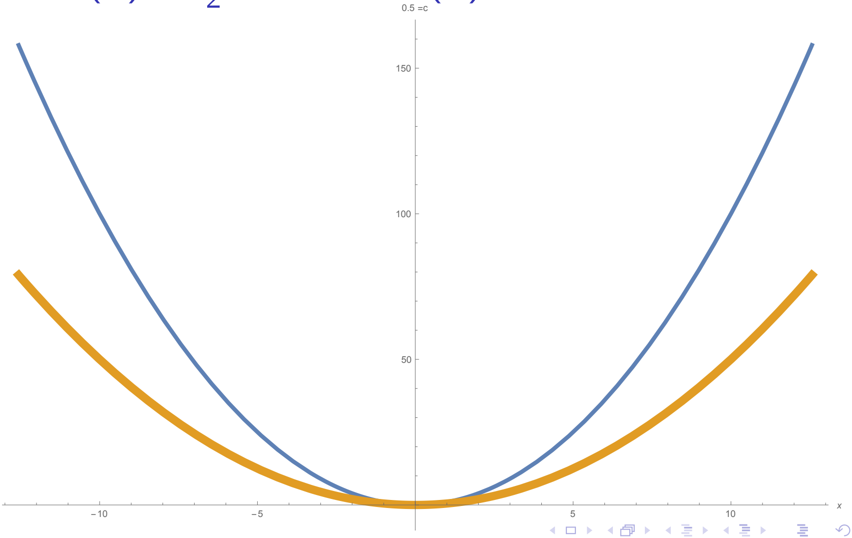
Gold: $f(x) = x^2$, Blue: $f(x) = x^2$



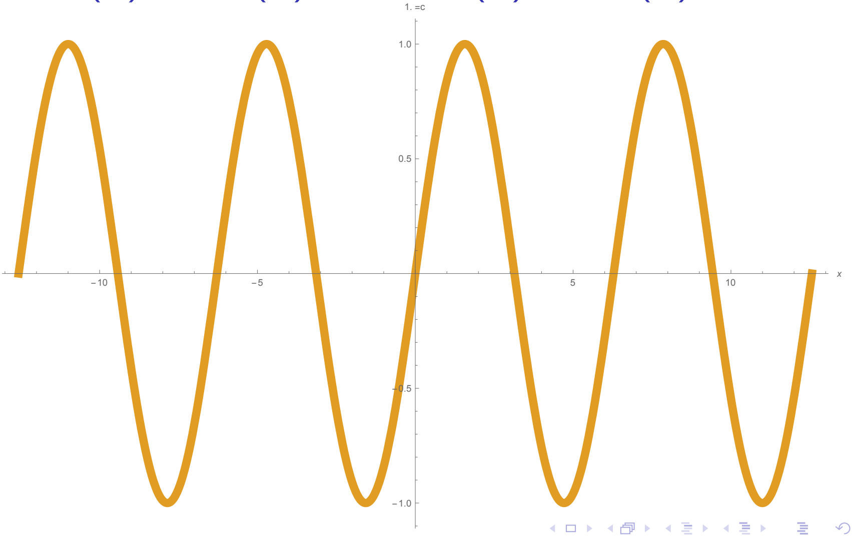
Gold: $f(x) = 3x^2$, Blue: $f(x) = x^2$



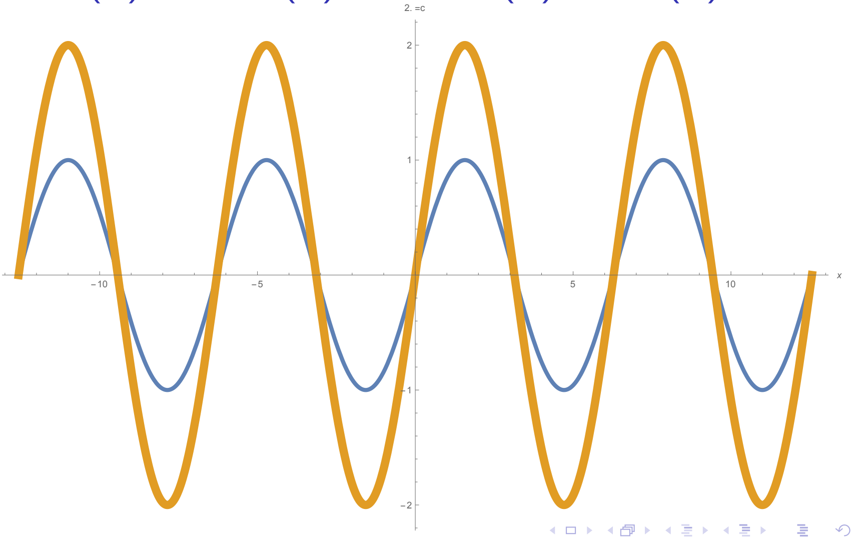
Gold: $f(x) = \frac{1}{2}x^2$, Blue: $f(x) = x^2$



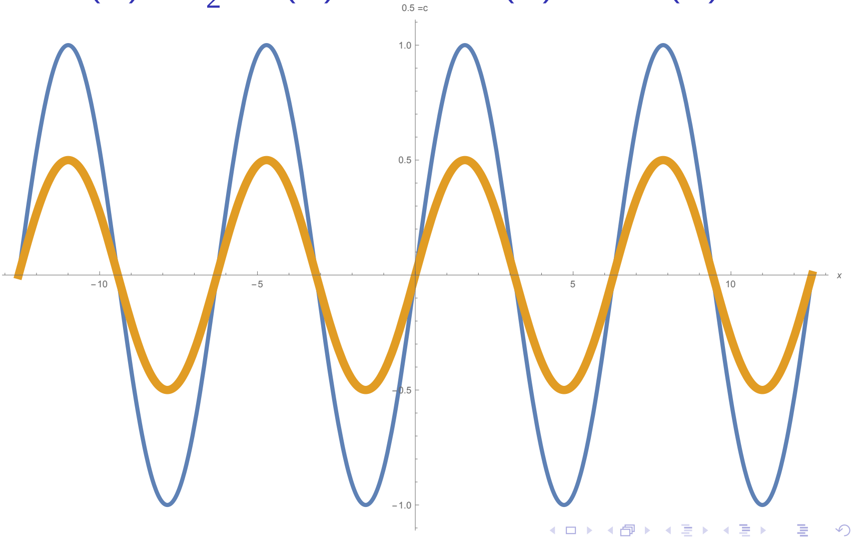
Gold: $f(x) = \sin(x)$, Blue: $f(x) = \sin(x)$



Gold: $f(x) = 2\sin(x)$, Blue: $f(x) = \sin(x)$



Gold: $f(x) = \frac{1}{2}\sin(x)$, Blue: $f(x) = \sin(x)$



Transformations (New)

Horizontal Stretch/Compression:

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$$\begin{cases} f(cx), c > 1 \quad : \\ \\ \\ \end{cases}$$

Transformations (New)

Horizontal Stretch/Compression:

$$\begin{cases} f(cx), c > 1 & \text{: compress horizontally by a factor of } 1/c \\ & \quad \text{units toward the } y\text{-axis} \\ & \end{cases}$$

Transformations (New)

Horizontal Stretch/Compression:

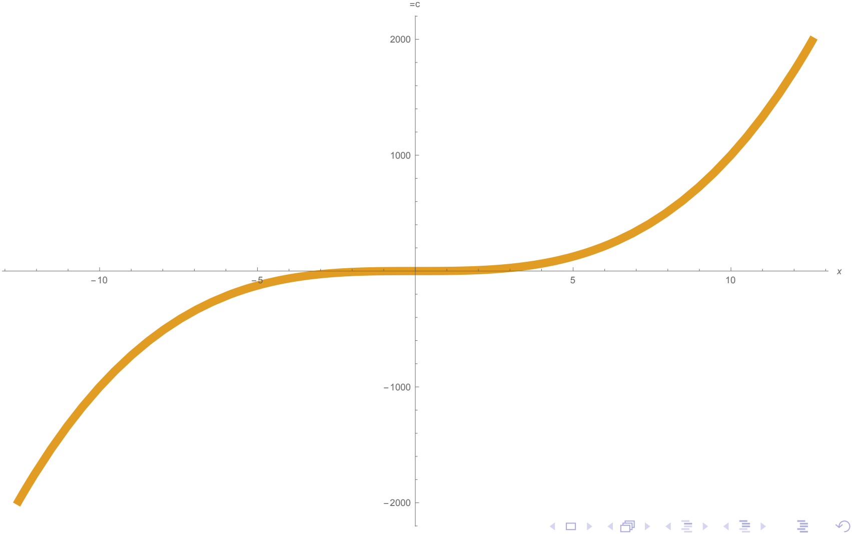
$$\begin{cases} f(cx), c > 1 & : \text{compress horizontally by a factor of } 1/c \\ & \quad \text{units toward the } y\text{-axis} \\ \\ f(cx), 0 < c < 1 & : \\ \\ \end{cases}$$

Transformations (New)

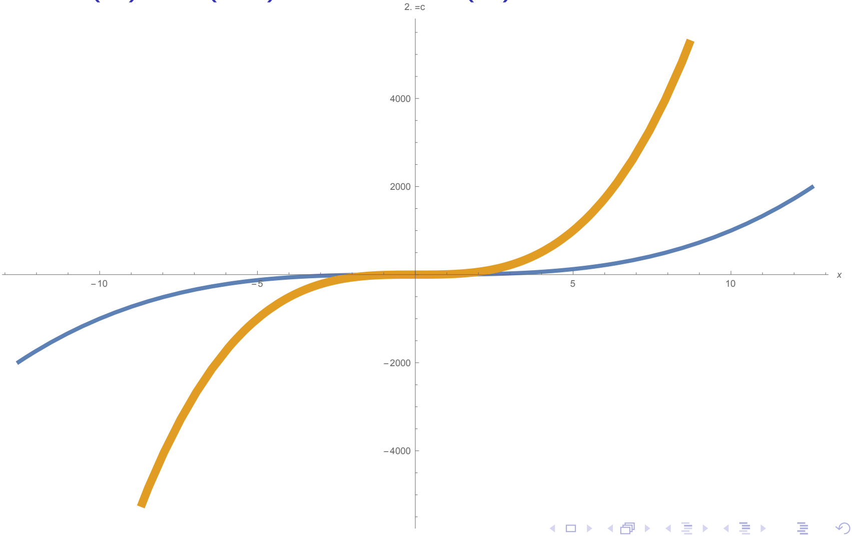
Horizontal Stretch/Compression:

$$\begin{cases} f(cx), c > 1 & : \text{compress horizontally by a factor of } 1/c \\ & \quad \text{units toward the } y\text{-axis} \\ \\ f(cx), 0 < c < 1 & : \text{stretch horizontally by a factor of } 1/c \\ & \quad \text{units away from the } y\text{-axis} \end{cases}$$

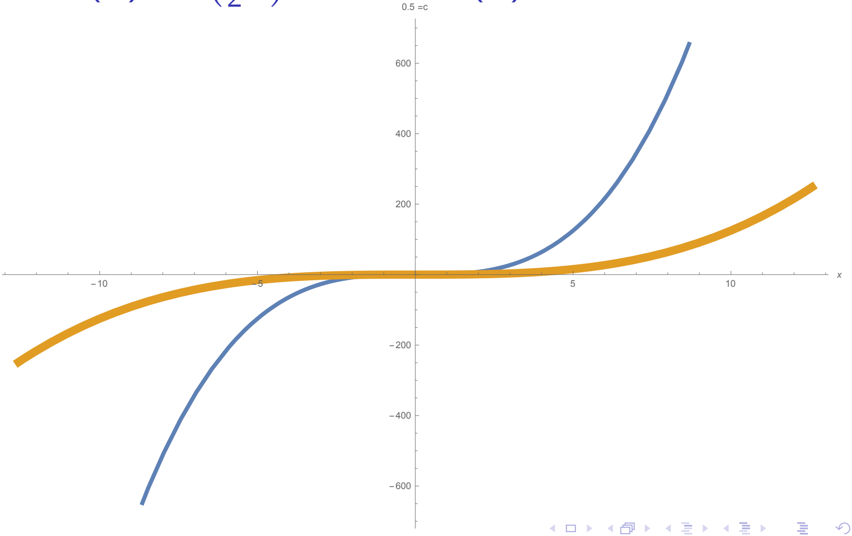
Gold: $f(x) = x^3$, Blue: $f(x) = x^3$



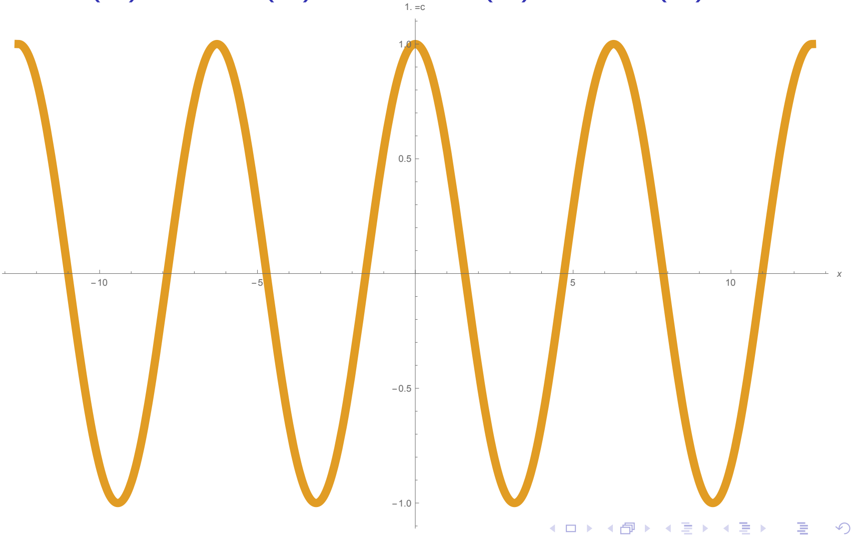
Gold: $f(x) = (2x)^3$, Blue: $f(x) = x^3$



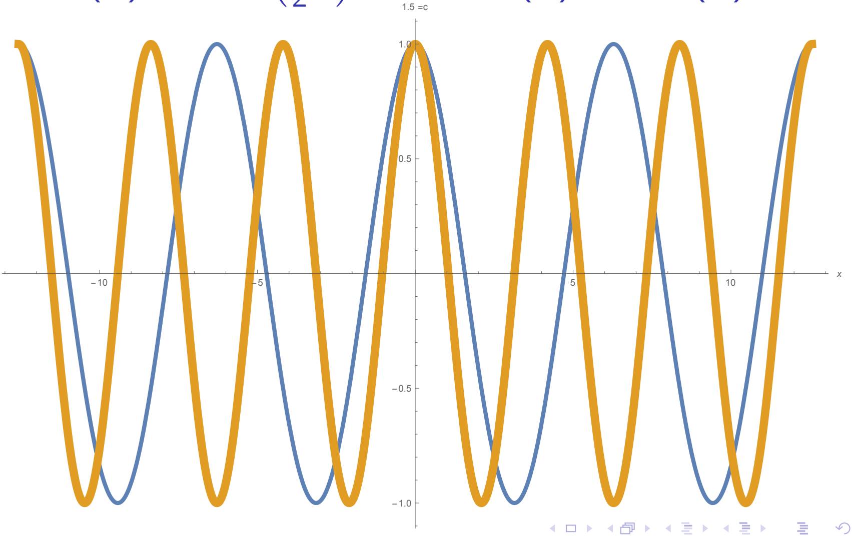
Gold: $f(x) = \left(\frac{1}{2}x\right)^3$, Blue: $f(x) = x^3$



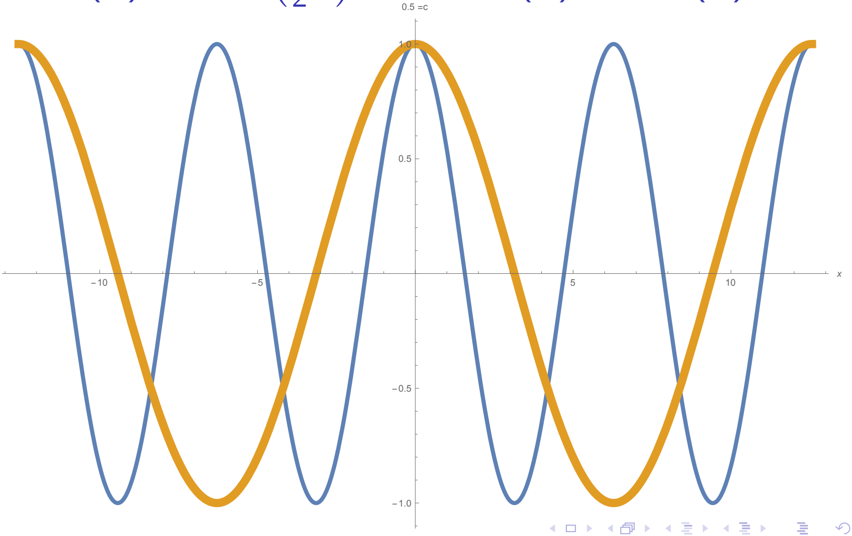
Gold: $f(x) = \cos(x)$, Blue: $f(x) = \cos(x)$



Gold: $f(x) = \cos\left(\frac{3}{2}x\right)$, Blue: $f(x) = \cos(x)$



Gold: $f(x) = \cos\left(\frac{1}{2}x\right)$, Blue: $f(x) = \cos(x)$



Example: If $f(-5) = 8$, give coordinates of the point for

1. $f(x+3)$
2. $f(x-3)$
3. $f(x)+3$
4. $f(x)-3$
5. $-3f(x)$
6. $\frac{-1}{3}f(x)$
7. $f(-3x)$
8. $f\left(\frac{-1}{3}x\right)$
9. $-f(x+2)$
10. $f(x-2)+3$

Example: If $f(3) = -2$, give coordinates of the point for

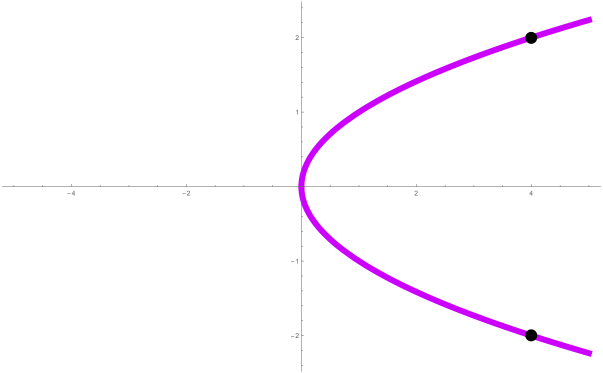
1. $f(x+2)$
2. $f(x-2)$
3. $f(x)+2$
4. $f(x)-2$
5. $5f(x)$
6. $f(5x)$
7. $\frac{1}{5}f(x)$
8. $f\left(\frac{1}{5}x\right)$
9. $-f(x+4)-2$
10. $f(x-4)+2$

Example: If $f(3) = -2$, give coordinates of the point for

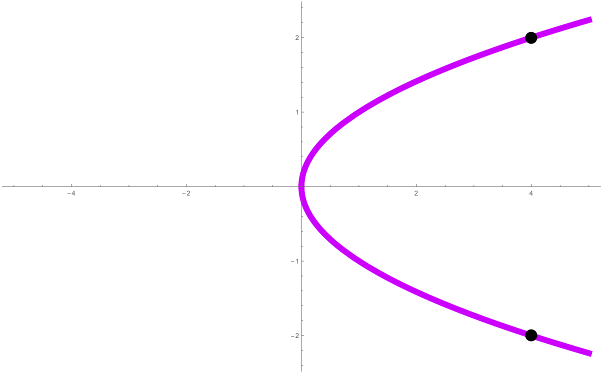
1. $f(x+2)$ $(1, -2)$
2. $f(x-2)$ $(5, -2)$
3. $f(x)+2$ $(3, 0)$
4. $f(x)-2$ $(3, -4)$
5. $5f(x)$ $(3, -10)$
6. $f(5x)$ $(3/5, -2)$
7. $\frac{1}{5}f(x)$ $(3, -2/5)$
8. $f\left(\frac{1}{5}x\right)$ $(15, -2)$
9. $-f(x+4)-2$ $(-1, 0)$
10. $f(x-4)+2$ $(7, 0)$

Symmetry with respect to the x-axis

Symmetry with respect to the x-axis

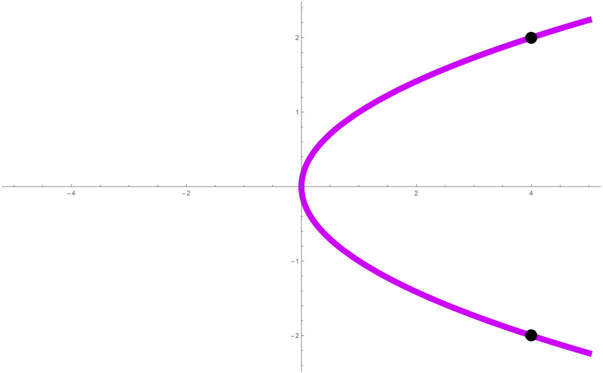


Symmetry with respect to the x-axis



Replace y with $-y$ and see no change.

Symmetry with respect to the x-axis



Replace y with $-y$ and see no change.
Reflect across the x-axis.

Symmetry with respect to the y-axis

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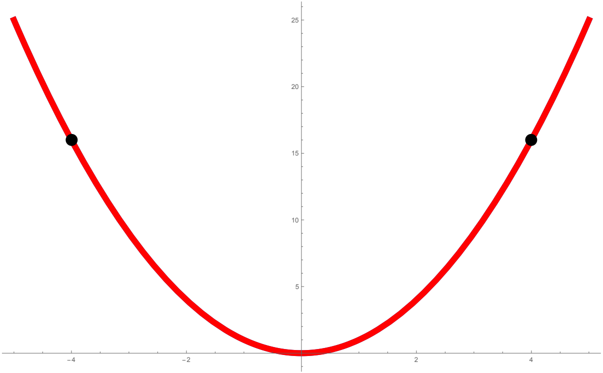
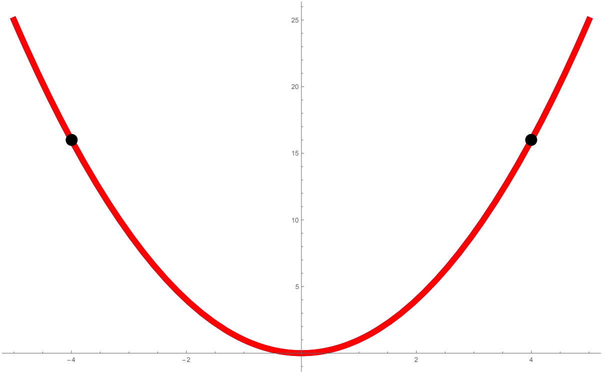
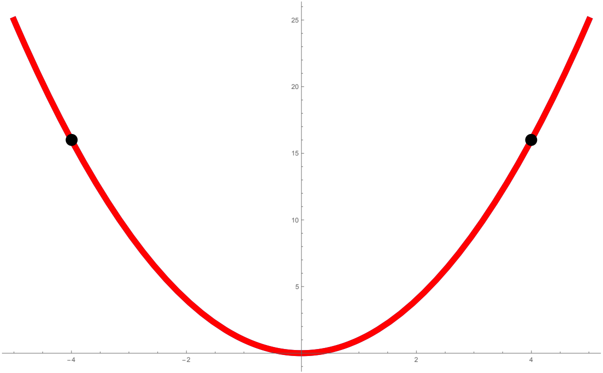


Symmetry with respect to the y-axis



Replace x with $-x$ and see no change.

Symmetry with respect to the y-axis



Replace x with $-x$ and see no change.
Reflect across the y-axis.

Symmetry with respect to the origin

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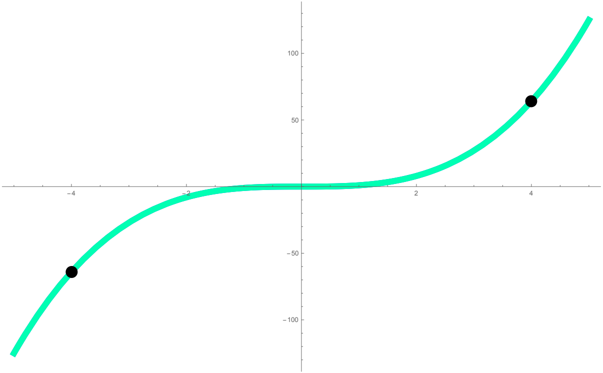
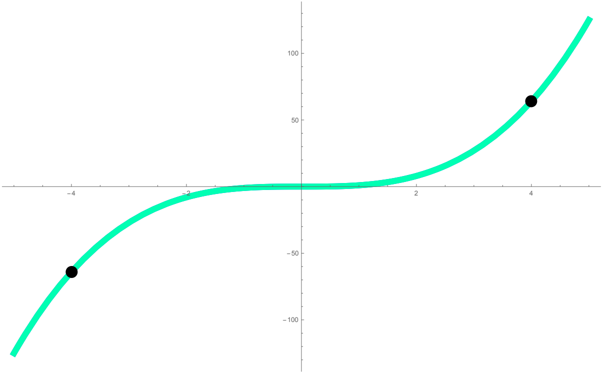
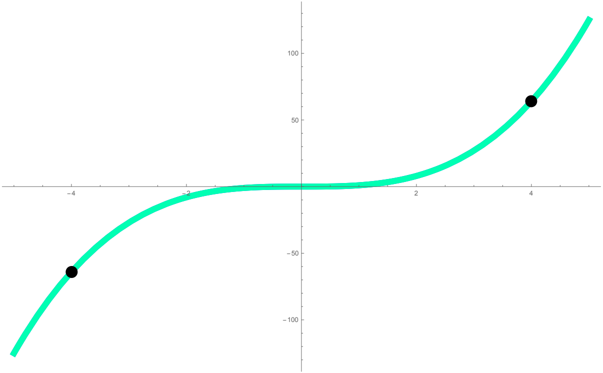


Symmetry with respect to the origin



Replace (x, y) with $(-x, -y)$ and see no change.

Symmetry with respect to the origin



Replace (x, y) with $(-x, -y)$ and see no change.
Reflect across the x-axis and the y-axis.

Example: A line segment connects the points $(1, 4)$ and $(3, 1)$. Reflect this segment across the

1. x-axis
2. y-axis
3. origin

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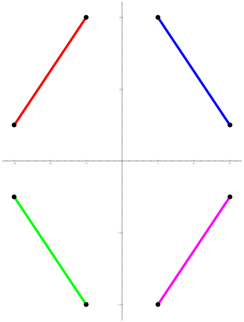

Even and Odd Functions

Even and Odd Functions

Even Functions:

Even and Odd Functions

Even Functions: Functions symmetric with respect to the y-axis.

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Examples:

Even and Odd Functions

Even Functions: Functions symmetric with respect to the y-axis.

Examples: $y = 1$,

Even and Odd Functions

Even Functions: Functions symmetric with respect to the y-axis.

Examples: $y = 1$, $y = x^2$,

Even and Odd Functions

Even Functions: Functions symmetric with respect to the y-axis.

Examples: $y = 1$, $y = x^2$, $y = x^4$,

Even and Odd Functions

Even Functions: Functions symmetric with respect to the y-axis.

Examples: $y = 1$, $y = x^2$, $y = x^4$, $y = x^{\text{even}}$,

Even and Odd Functions

Even Functions: Functions symmetric with respect to the y-axis.

Examples: $y = 1$, $y = x^2$, $y = x^4$, $y = x^{\text{even}}$, $y = \cos x$,

Even and Odd Functions

Even Functions: Functions symmetric with respect to the y-axis.

Examples: $y = 1$, $y = x^2$, $y = x^4$, $y = x^{\text{even}}$, $y = \cos x$, $f(x) = f(-x)$

Even and Odd Functions

Even Functions: Functions symmetric with respect to the y-axis.

Examples: $y = 1$, $y = x^2$, $y = x^4$, $y = x^{\text{even}}$, $y = \cos x$, $f(x) = f(-x)$

Odd Functions:

Even and Odd Functions

Even Functions: Functions symmetric with respect to the y-axis.

Examples: $y = 1$, $y = x^2$, $y = x^4$, $y = x^{\text{even}}$, $y = \cos x$, $f(x) = f(-x)$

Odd Functions: Functions symmetric with respect to the origin.

Even and Odd Functions

Even Functions: Functions symmetric with respect to the y-axis.

Examples: $y = 1$, $y = x^2$, $y = x^4$, $y = x^{\text{even}}$, $y = \cos x$, $f(x) = f(-x)$

Odd Functions: Functions symmetric with respect to the origin.

Examples:

Even and Odd Functions

Even Functions: Functions symmetric with respect to the y-axis.

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$f(x) = f(-x)$

Odd Functions: Functions symmetric with respect to the origin.

Examples: $y = x$,

Even and Odd Functions

Even Functions: Functions symmetric with respect to the y-axis.

Examples: $y = 1$, $y = x^2$, $y = x^4$, $y = x^{\text{even}}$, $y = \cos x$, $f(x) = f(-x)$

Odd Functions: Functions symmetric with respect to the origin.

Examples: $y = x$, $y = x^3$,

Even and Odd Functions

Even Functions: Functions symmetric with respect to the y-axis.

Examples: $y = 1$, $y = x^2$, $y = x^4$, $y = x^{\text{even}}$, $y = \cos x$,
$f(x) = f(-x)$

Odd Functions: Functions symmetric with respect to the origin.

Examples: $y = x$, $y = x^3$, $y = x^{\text{odd}}$,

Even and Odd Functions

Even Functions: Functions symmetric with respect to the y-axis.

Examples: $y = 1$, $y = x^2$, $y = x^4$, $y = x^{\text{even}}$, $y = \cos x$, $f(x) = f(-x)$

Odd Functions: Functions symmetric with respect to the origin.

Examples: $y = x$, $y = x^3$, $y = x^{\text{odd}}$, $y = \sin x$,

Even and Odd Functions

Even Functions: Functions symmetric with respect to the y-axis.

Examples: $y = 1$, $y = x^2$, $y = x^4$, $y = x^{\text{even}}$, $y = \cos x$,
$f(x) = f(-x)$

Odd Functions: Functions symmetric with respect to the origin.

Examples: $y = x$, $y = x^3$, $y = x^{\text{odd}}$, $y = \sin x$,
$f(-x) = -f(x)$

Determine Symmetry, Even/Odd (if appropriate)

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1. $y = 4 - x^2$

Determine Symmetry, Even/Odd (if appropriate)

1. $y = 4 - x^2$

2. $y = -x$

Determine Symmetry, Even/Odd (if appropriate)

1. $y = 4 - x^2$
2. $y = -x$
3. $|y| = 2x - 4$

Determine Symmetry, Even/Odd (if appropriate)

1. $y = 4 - x^2$

2. $y = -x$

3. $|y| = 2x - 4$

4. $y = \dfrac{-1}{x}$

Determine Whether Even, Odd, or Neither

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1. $f(x) = 4x^3 - 2x$

Determine Whether Even, Odd, or Neither

1. $f(x) = 4x^3 - 2x$
2. $f(x) = -3x^4 + 7x^2 - 15$

Determine Whether Even, Odd, or Neither

1. $f(x) = 4x^3 - 2x$
2. $f(x) = -3x^4 + 7x^2 - 15$
3. $f(x) = 8x^3 - 3x^2$

Determine Whether Even, Odd, or Neither

1. $f(x) = 4x^3 - 2x$
2. $f(x) = -3x^4 + 7x^2 - 15$
3. $f(x) = 8x^3 - 3x^2$
4. $f(x) = \sqrt{x^2 + 4}$

Determine Whether Even, Odd, or Neither

1. $f(x) = 4x^3 - 2x$
2. $f(x) = -3x^4 + 7x^2 - 15$
3. $f(x) = 8x^3 - 3x^2$
4. $f(x) = \sqrt{x^2 + 4}$
5. $f(x) = 3x^2 + 4x - 2$

Determine Whether Even, Odd, or Neither

1. $f(x) = 4x^3 - 2x$
2. $f(x) = -3x^4 + 7x^2 - 15$
3. $f(x) = 8x^3 - 3x^2$
4. $f(x) = \sqrt{x^2 + 4}$
5. $f(x) = 3x^2 + 4x - 2$
6. $f(x) = 24x^7 + 4x$

Thank you for your attention.