

Cooperative and Competitive Species

Below are two systems of nonlinear differential equations, one modeling cooperative species and one competitive. A cooperative system is one in which both species benefit from interaction with one another, such as flowers and bees. A competitive system is one in which interaction between the two species is detrimental to both, such as automobiles and pedestrians. We wish to understand both systems for all nonnegative initial conditions (starting populations); for this reason, consider only $x \geq 0$ and $y \geq 0$. For each system:

- Identify the system as cooperative or competitive.
- Find and classify the equilibria.
- Draw a neat, labelled phase plane, including clearly identified equilibria and separatrices, and arrows of flow.

$$\text{A. } \begin{cases} \frac{dx}{dt} = -5x + 3xy \\ \frac{dy}{dt} = -3y + 2xy \end{cases}$$

$$\text{B. } \begin{cases} \frac{dx}{dt} = 9x - 2x^2 - 4xy \\ \frac{dy}{dt} = 8y - 5xy - 3y^2 \end{cases}$$