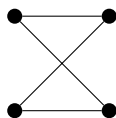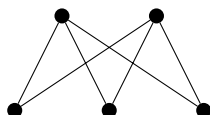# Planar Graphs

Definition: A graph is <u>planar</u> if we can draw it on the plane (a flat surface) with no crossing edges. If the graph is actually drawn in such a way, we say that the drawing is a <u>plane graph</u>.
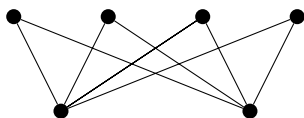
Example:
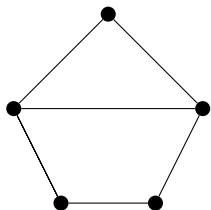
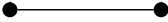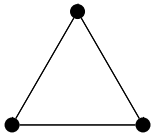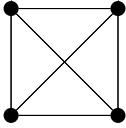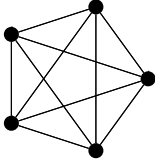**Example:**

Example:

Example: What about paths, cycles, wheels, trees? What about complete graphs? See page 2.

Definition: If G is a connected plane graph, it divides the plane into different regions, which are called <u>faces</u>. Note: The "outside" of a graph is always one face.

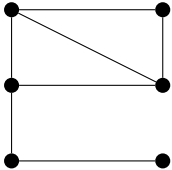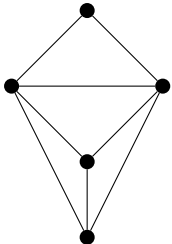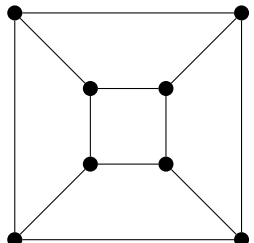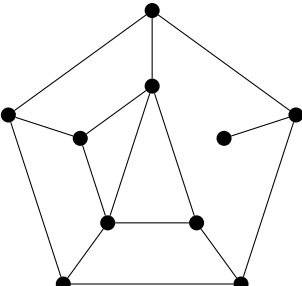**Example:**

1. Try to draw each of the following complete graphs as a plane graph.

| K_i	Drawn as a plane graph	Is K_i planar?
$K_1 =$		
$K_2 =$		
$K_3 =$		
$K_4 =$		
$K_5 =$		
K_n		

2. For each of the following plane graphs, count the number of vertices, edges, and faces.

Graph	V	E	F	

3. Do you see any pattern emerge among V, E, and F for these plane graphs?
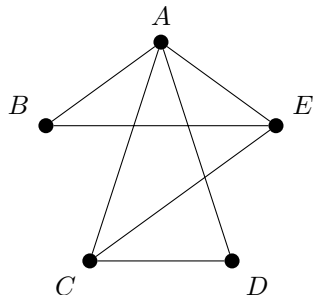
Homework Assignment — Due Monday, November 1

1. For the following plane graph, count the number of vertices, the number of edges, and the number of faces. Verify that Euler's formula holds.
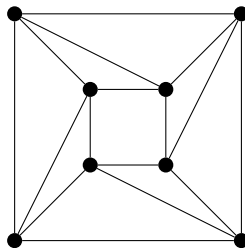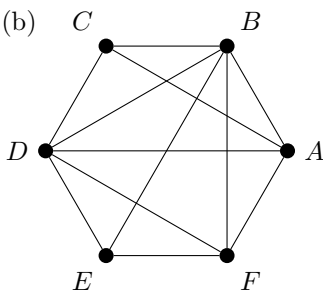
2. For each of the following graphs, if the graph is planar, draw it in the plane so that no edges cross. Otherwise, state that the graph is not planar.
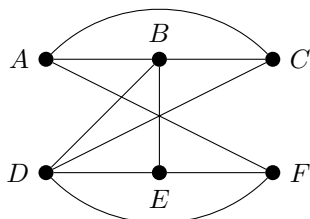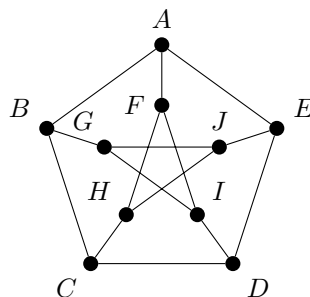
   (a)

   (b)

   (c)

   (d)

3. Let G be a connected planar graph. Explain why, no matter how we draw G as a plane graph, the number of faces of G will always be the same.

4. Suppose G is a connected plane graph with nine vertices, where the degrees of the vertices are 2, 2, 2, 3, 3, 3, 4, 4, and 5. How many edges does G have? How many faces does G have?

5. Suppose you have a graph G that is *not* planar. Is there a way to make it planar by adding more vertices and edges? If so, describe your method; if not, why not?

6. Suppose you have a graph G that is planar. Is there a way to make it not planar by adding more vertices and edges? If so, describe your method; if not, why not?