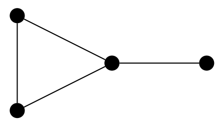
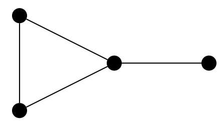
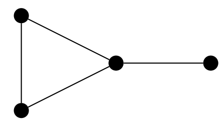
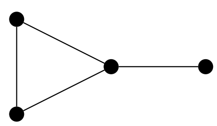
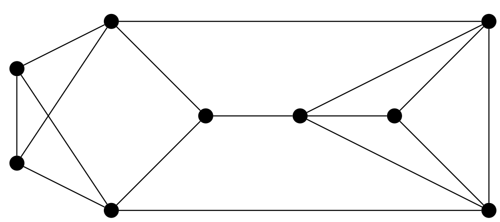
Graph Colorings

Definition: A (vertex) coloring of a graph is an assignment of colors to the vertices of the graph so that vertices that are joined by an edge (adjacent vertices) have different colors.

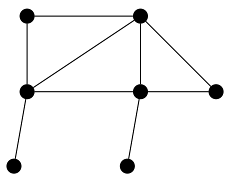
Example:   

Definition: For a graph G, the smallest number of colors needed to color G is called the chromatic number of G and is denoted $\chi(G)$. Note that χ is the Greek letter chi, for chromatic.

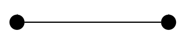
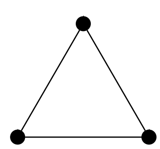
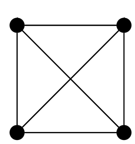
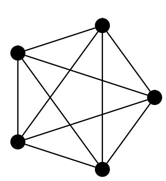
Example: If G is  , then $\chi(G) = 3$. It is possible to color G using just 3 colors, and we need at least 3 colors because G has a triangle.

Example: If G is  find $\chi(G)$.

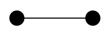
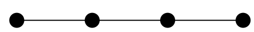
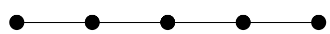
1. Color each of the vertices of the following graph red (R), white (W), or blue (B) in such a way that no adjacent vertices have the same color.



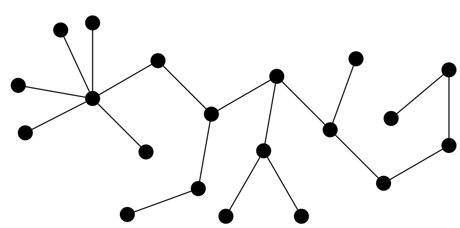
2. What is the coloring number of the following complete graphs?

K_i	$\chi(K_i)$
 $K_1 =$	
 $K_2 =$	
$K_3 =$ 	
$K_4 =$ 	
$K_5 =$ 	
K_n	

3. What is the coloring number of the following paths?

P_i	$\chi(P_i)$
$P_1 =$ 	
$P_2 =$ 	
$P_3 =$ 	
$P_4 =$ 	
$P_5 =$ 	
P_n	

4. (a) Find the chromatic number of the following tree:



(b) Draw a tree with 11 vertices, and find its chromatic number.

5. Corncob College elects 10 students to serve as officers on 8 committees. The list of the members of each of the committees is:

   - Corn Feed Committee: Darcie, Barb, Kyler
   - Dorm Policy Committee: Barb, Jack, Anya, Kaz
   - Extracurricular Committee: Darcie, Jack, Miranda
   - Family Weekend Committee: Kyler, Miranda, Jenna, Natalie
   - Homecoming Committee: Barb, Jenna, Natalie, Skye
   - Off Campus Committee: Kyler, Jenna, Skye
   - Parking Committee: Jack, Anya, Miranda
   - Student Fees Committee: Kaz, Natalie

   They need to schedule meetings for each of these committees, but two committees cannot meet at the same time if they have any members in common.

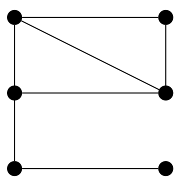
   (a) Draw a graph representing this situation.
   (Hint: let the vertices represent the committees.)

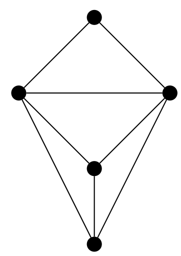
   (b) How many different meeting times will we need?

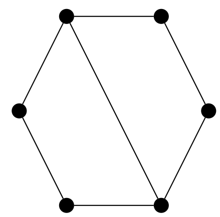
Math 105
Homework Assignment — Due Friday, October 29

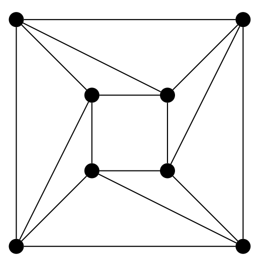
Book Problems: None

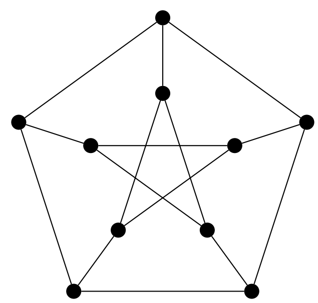
1. Find the chromatic number for each of the following graphs.

(a) 

(b) 

(c) 

(d) 

(e) 

2. Find $\chi(C_n)$ and $\chi(W_n)$ for every integer $n \geq 3$, where C_n denotes the cycle with n vertices and W_n denotes the wheel with n spokes ($n + 1$ vertices). See examples on Page 6. Your answers will change as n changes.

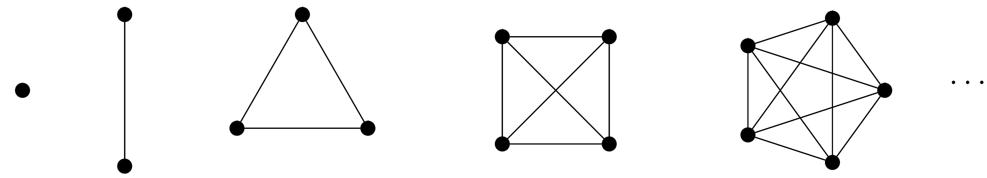
3. The mathematics department at Cornucopia College will offer seven courses next semester: Math 105 (M), Numerical Analysis (N), Linear Operators (O), Probability (P), Differential Equations (Q), Real Analysis (R), Statistics (S). The department has twelve students, who will take the following classes:

Alice:	N, O, Q	Dan:	N, O	Greg:	M, P	Jill:	N, S, Q
Bob:	N, R, S	Emma:	O, M	Homer:	R, O	Kate:	P, S
Chaz :	R, M	Fonz:	N, R	Inez:	N, Q	Lara:	P, Q

The department needs to schedule class times for each of these courses, but two courses cannot meet at the same time if they have any students in common. How many different class times will they need?

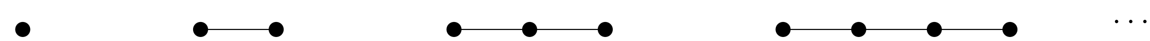
Some Important Graphs

Complete Graphs



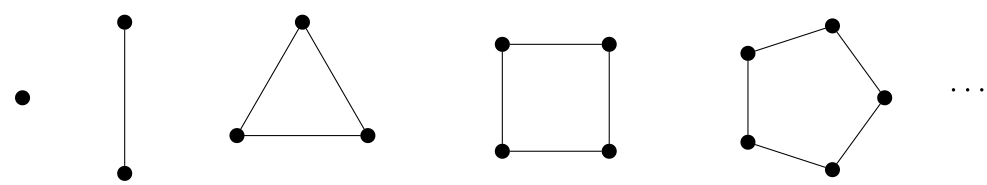
K_1 K_2 K_3 K_4 K_5

Paths



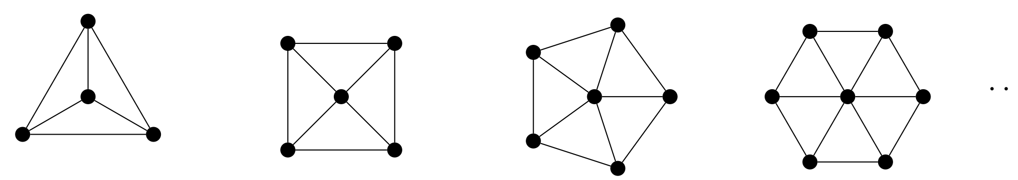
P_1 P_2 P_3 P_4

Cycles



C_1 C_2 C_3 C_4 C_5

Wheels



W_3 W_4 W_5 W_6

Trees

Connected graphs with no circuits