Paths and Circuits

<u>Path</u>: a sequence of adjacent edges, where the edges used are used only once.

<u>Length</u>: the number of edges in the path.

<u>Connected</u>: a graph in which a path exists between any two vertices.

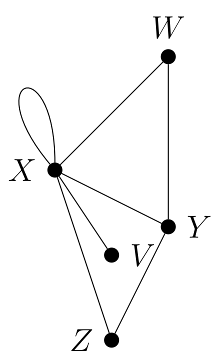
<u>Disconnected</u>: not connected!

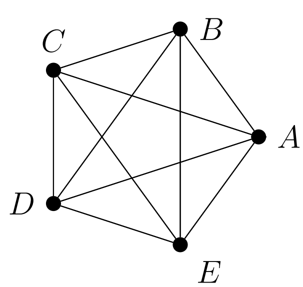
<u>Components</u>: connected parts of the graph.

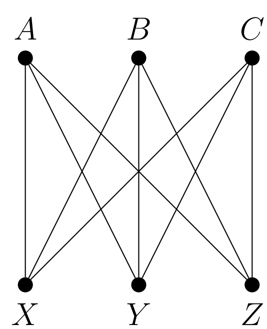
<u>Circuit</u>: a path that starts and ends at the same vertex.

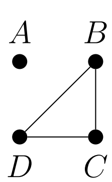
<u>Euler Path</u>: a path in a connected graph that passes through every edge of the graph once and only once.

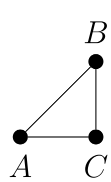
<u>Euler Circuit</u>: a path in a connected graph that starts and ends at the same vertex, and passes through every edge of the graph once and only once.

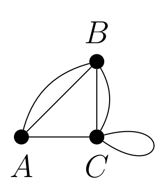


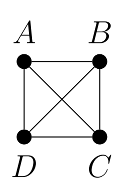


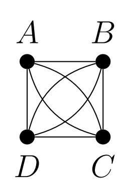




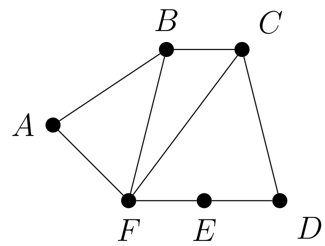




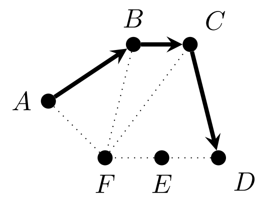




1. There are nine different paths from A to D in the following graph. Find them all.



For example, one path is $ABCD$, as shown:

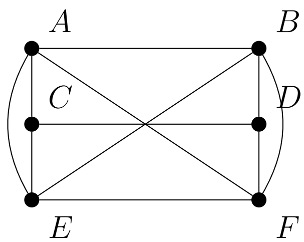
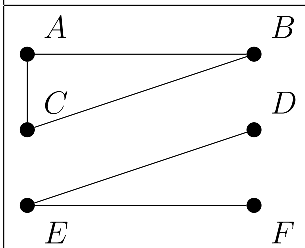
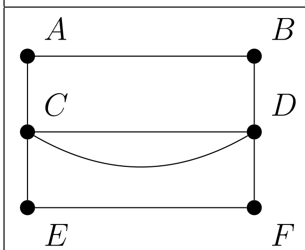
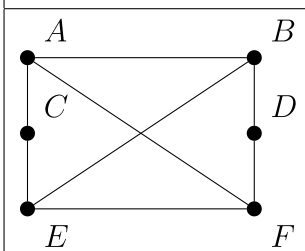


2. Consider the following graphs.

   (a) Write the degree list for the graph.

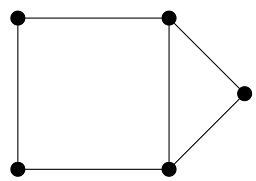
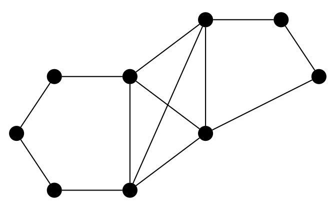
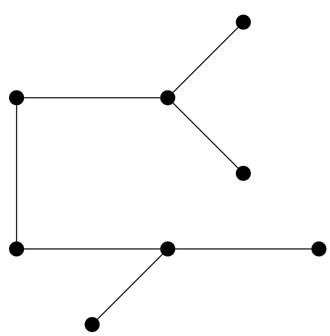
   (b) Is the graph connected? If so, is there an Euler path (path that uses every edge exactly once)?

   (c) Is the graph connected? If so, is there an Euler circuit (circuit that uses every edge exactly once)?

	**Euler Path:** **Euler Circuit:**	**Degree List:**
	**Euler Path:** **Euler Circuit:**	**Degree List:**
	**Euler Path:** **Euler Circuit:**	**Degree List:**
	**Euler Path:** **Euler Circuit:**	**Degree List:**

3. Do you see a pattern between whether such paths or circuits exist and what numbers are in the degree lists?

4. For each of the following graphs, calculate the degree list. Then use the degree list to determine whether it has an Euler path or an Euler circuit or neither.

Graph	Degree list	Euler path?	Euler circuit?
			
			
			

For connected graphs, if there are no odd vertices then there is an Euler circuit (and thus an Euler path as well). If there are exactly two odd vertices, there is an Euler path but not an Euler circuit. Otherwise, there is neither an Euler path nor an Euler circuit!