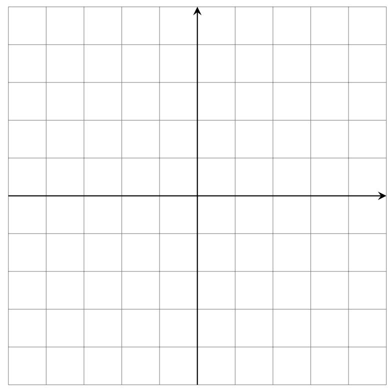
Taxicab Geometry Worksheets

Exploring Mathematics, Spring 2010

Day 1: Taxicab Distances

1. (a) Graph the points $A = (1, 3)$, $B = (1, -2)$, $C = (-3, -1)$, and $D = (0, 3)$.



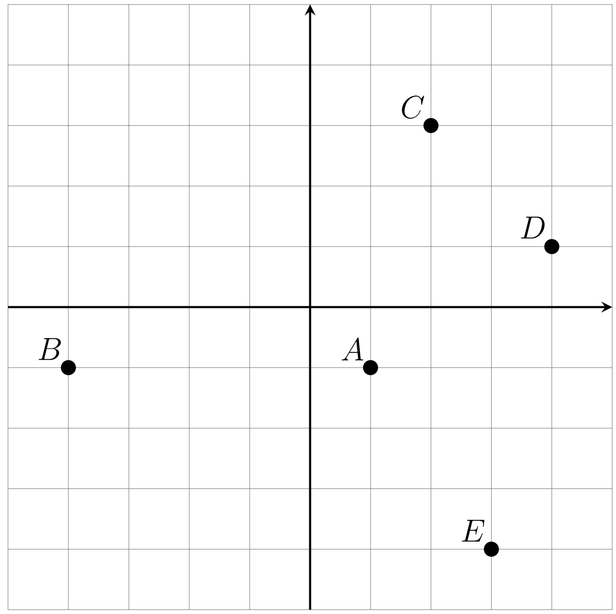
(b) Now find the following distances in both Euclidean and taxicab geometries. Give a decimal approximation to 2 decimal places.

	Euclidean distance	Taxicab distance
from A to B		
from B to C		
from C to D		

(c) If you know the Euclidean distance between two points, does that tell you what the taxicab distance is? Why or why not?

(d) If you know the taxicab distance between two points, does that tell you what the Euclidean distance is? Why or why not?

2. (a) Consider the points in the following graph:

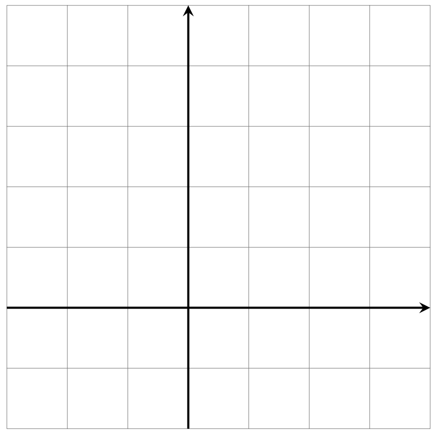


Calculate the following distances in both Euclidean and taxicab geometries. Give a decimal approximation to 2 decimal places.

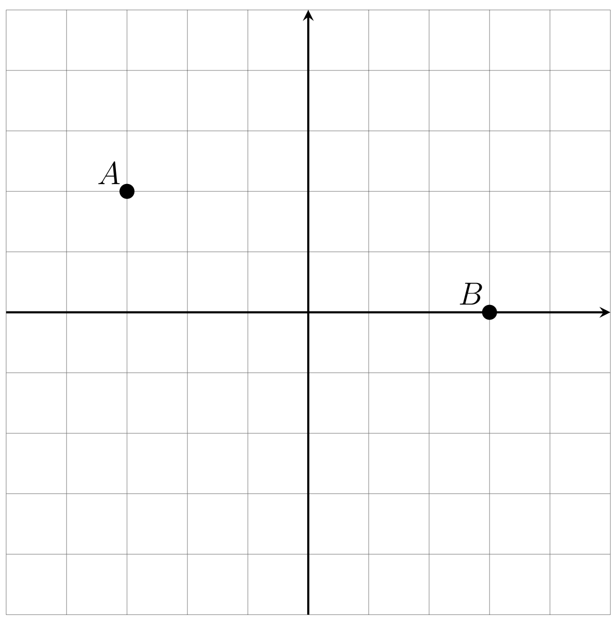
	Euclidean distance	Taxicab distance
from A to B		
from A to C		
from A to D		
from A to E		

(b) Is the Euclidean distance between two points always less than or equal to the taxicab distance? If so, explain why. If not, give an example where the Euclidean distance is greater than the taxicab distance.

3. One night the 911 dispatcher for Taxicab City receives a report of an accident at $X = (-1, 4)$. There are two police cars in the area, car C at $(2, 1)$ and car D at $(-1, -1)$. Which car should be sent to the scene of the accident to arrive most quickly? (Since the cars must drive on the streets, we use taxicab geometry to measure distances.)



4. Using taxicab geometry, consider the points $A = (-3, 2)$ and $B = (3, 0)$.



(a) Is the point $(-2, -3)$ closer to A or to B?

(b) Is the point $(1, -2)$ closer to A or to B?

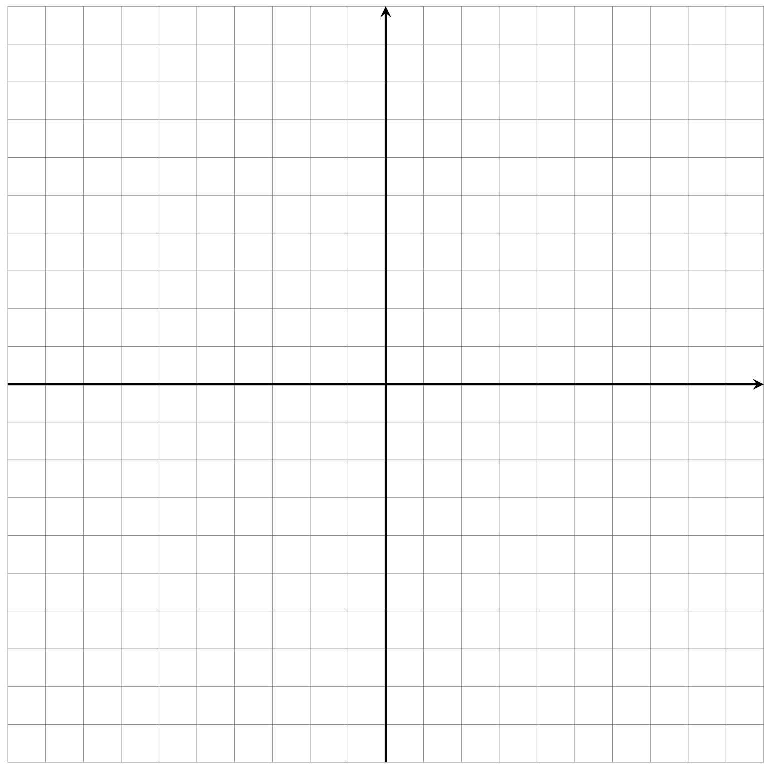
(c) Find one point that is exactly the same distance from A is it is from B. Mark it on the graph.

(d) Find another such point. Mark it on the graph.

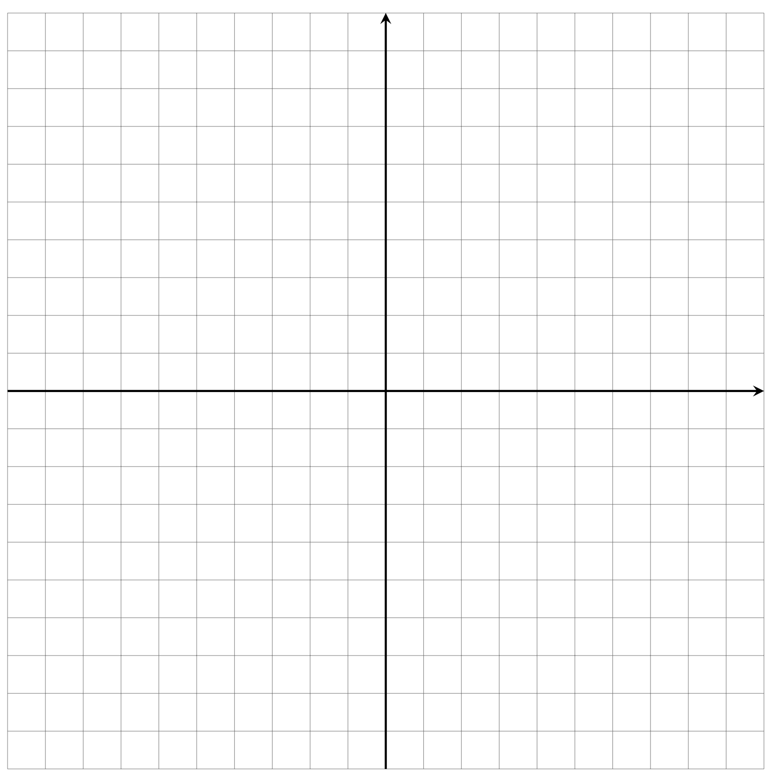
(e) Mark all points on the graph that are equally distant from A and from B. (Remember, this includes points with non-integer coordinates.)

Day 2: Taxicab Circles

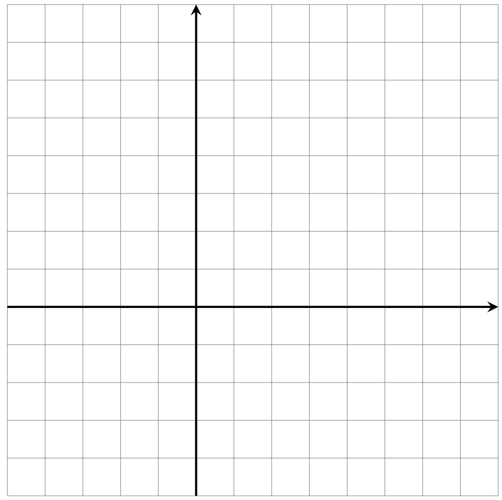
1. Draw the taxicab circle of radius 5 around the point $P = (3, 4)$.



2. Draw the taxicab circle of radius 6 around the point $Q = (2, -1)$.



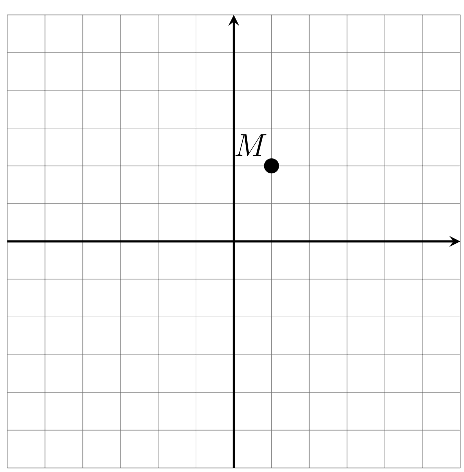
3. On a single graph, draw taxicab circles around point $R = (1, 2)$ of radii 1, 2, 3, and 4.



4. Describe a quick technique for drawing a taxicab circle of radius r around a point P.

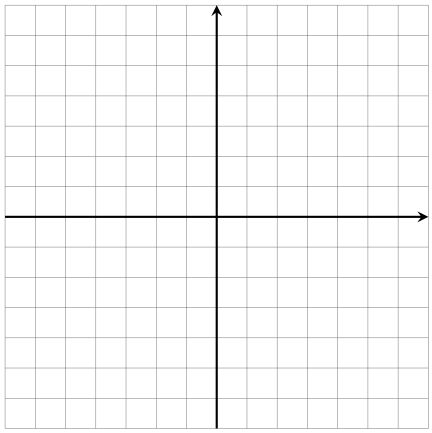
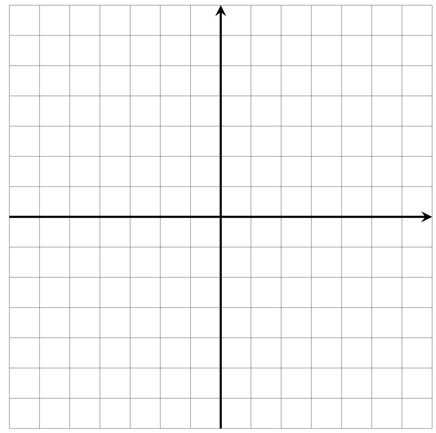
5. What is a good value for π in taxicab geometry?

6. George works in Taxicab City for the 3M plant, located at $M = (1, 2)$. He goes out to eat for lunch once a week, and out of company loyalty, he likes to walk exactly 3 blocks from the plant to do so. Where in the city are restaurants at which George can eat? Draw their locations on the graph.

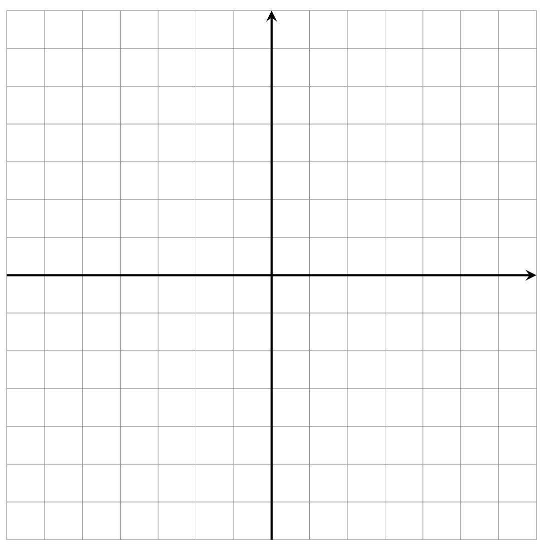


7. Fred Finnegan campaigned to become mayor of Taxicab City on a platform of installing drinking fountains throughout the city, so that no one would ever be more than three blocks from a free drink of water. He won the election, but since his predecessor depleted the city treasury, he needs to spend money judiciously. Suppose the city is 14 blocks square, as shown in the grids below.

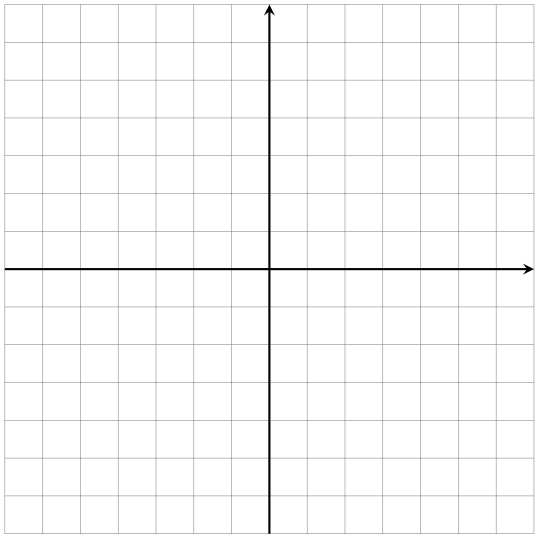
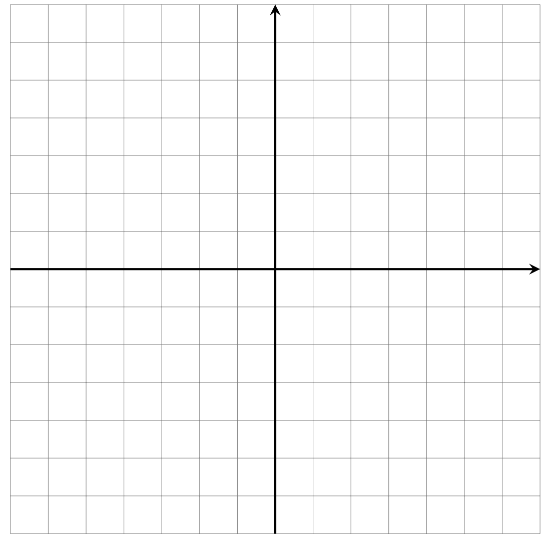
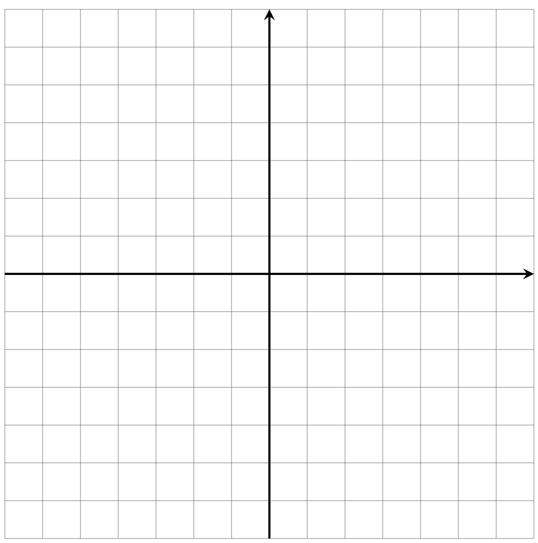
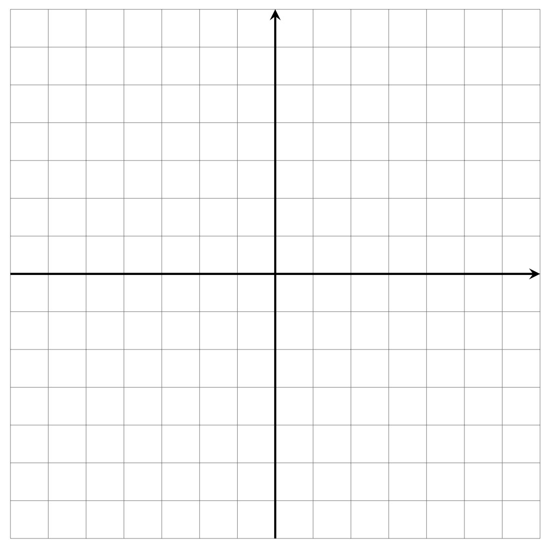
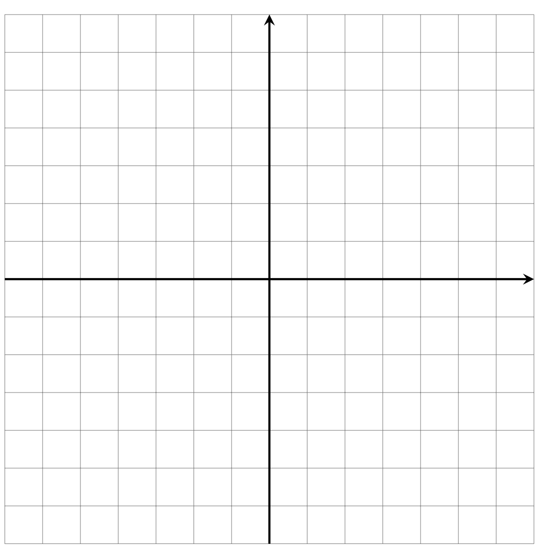
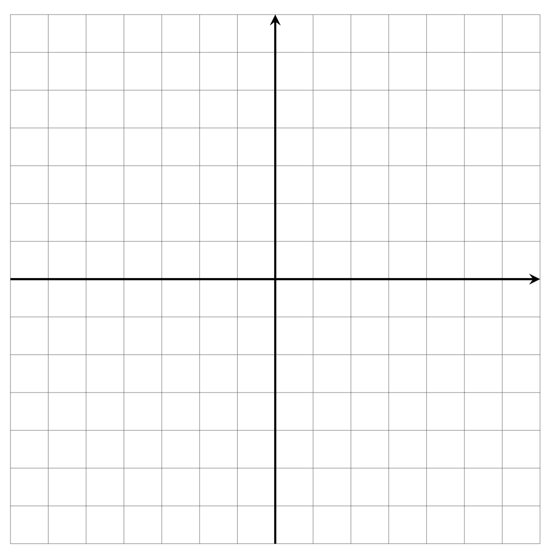
   (a) Come up with at least 2 different plans for where to locate the drinking fountains. (If you need more room to practice, there are more grids on the back of this page.)

   (b) Come up with the most cost-efficient way for the mayor to fulfill his campaign promise. (That is, give a map of where to locate the drinking fountains.)

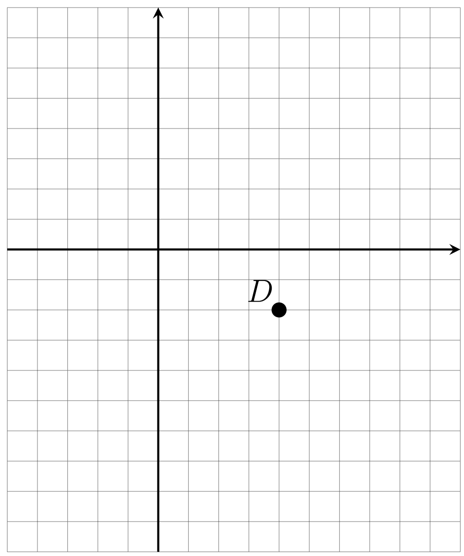
   

   (c) Suppose Taxicab City were to expand into the surrounding countryside as the population grows. Describe how to extend your pattern indefinitely into the new, surrounding blocks.

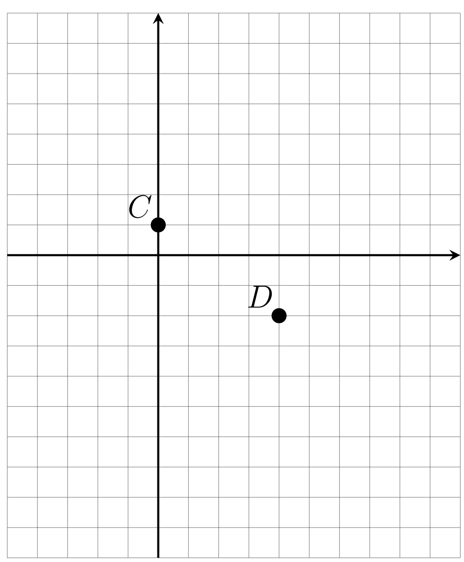







Day 3: Taxicab Applications

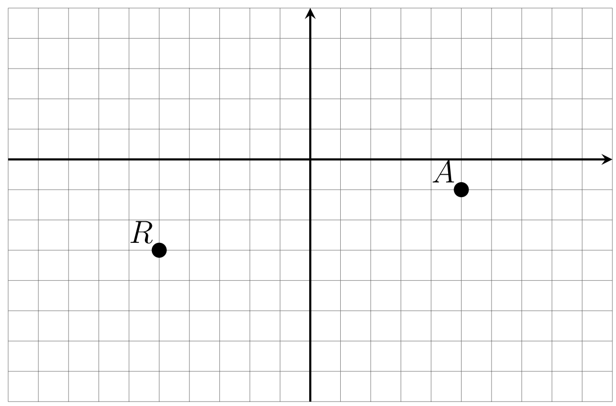
1. Doug moves to Taxicab City and works at the distillery at $D = (4, -2)$. Like Alice and Bob, he walks to work along the city blocks. Because of a heart condition, Doug cannot live more than 5 blocks from work. On a graph, shade in all the places Doug can live.



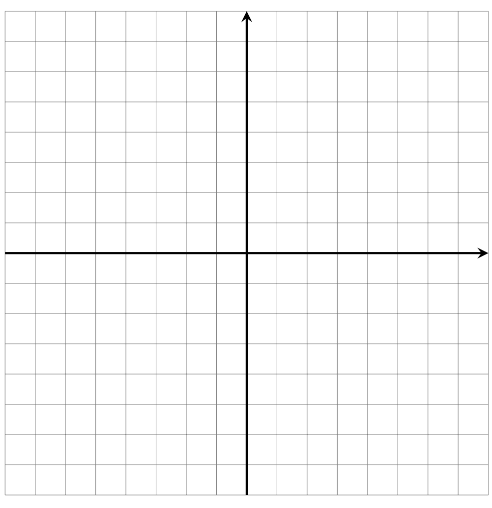
2. On second thought, Doug realizes that he also wants to live near the church at $C = (0, 1)$. He is looking for an apartment A so that the distance from A to C *plus* the distance from A to D is at most 9 blocks. Shade in all the places Doug can live.



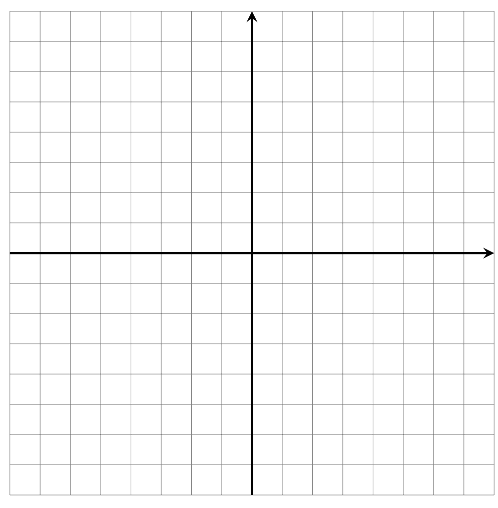
3. Acme Industrial Parts wants to build a factory in Taxicab City. It needs to receive shipments from the railroad depot at $R = (-5, -3)$ and ship parts out by plane, so it wants the factory to be located so that the total distance from the depot to the factory to the airport at $A = (5, -1)$ is at most 16 blocks. However, a city noise ordinance prohibits any factories from being built within 3 blocks of the public library at $L = (-4, 2)$. Where can Acme build its factory?



4. (a) Draw the taxicab ellipse with foci $(-2, 1)$ and $(4, 1)$, so that the total distance from each point to the foci is 8.
   (b) Draw the taxicab ellipse with foci $(-3, -2)$ and $(4, 3)$, so that the total distance from each point to the foci is 16.



(a)

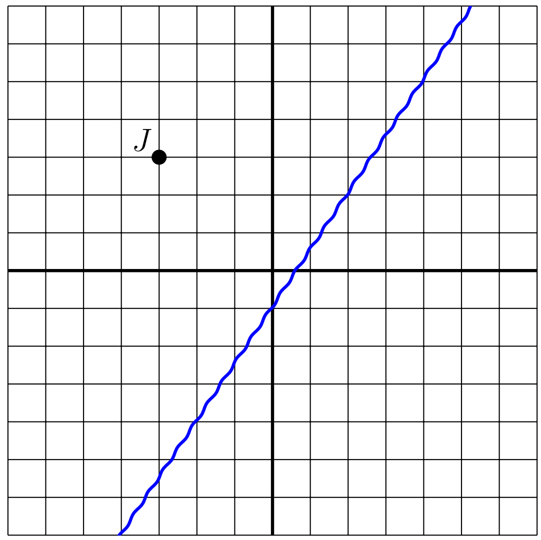


(b)

5. Describe how to draw a taxicab ellipse if you know the foci and the total distance from each point to the two foci.

Day 4: Taxicab Points and Lines

1. In nearby Omnibus City, a river runs through town on a line running through $(0, -1)$ and $(2, 2)$ as shown.

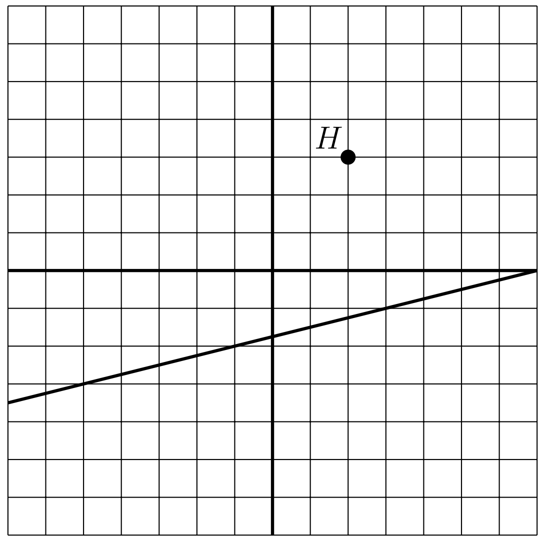


(a) Josephine currently lives in an apartment at $J = (-3, 3)$. What point on the river is closest to her apartment (in taxicab geometry, of course)?

(b) How far is Josephine's apartment from the river (in taxicab geometry, of course)?

(c) Josephine wants to move to a scenic apartment within three blocks' walk of the river. Where should Josephine look for an apartment?

2. The bike path in Taxicab City runs on a line through $(-5,-3)$ and $(3,-1)$, as shown. Hilda lives at $H = (2,3)$.

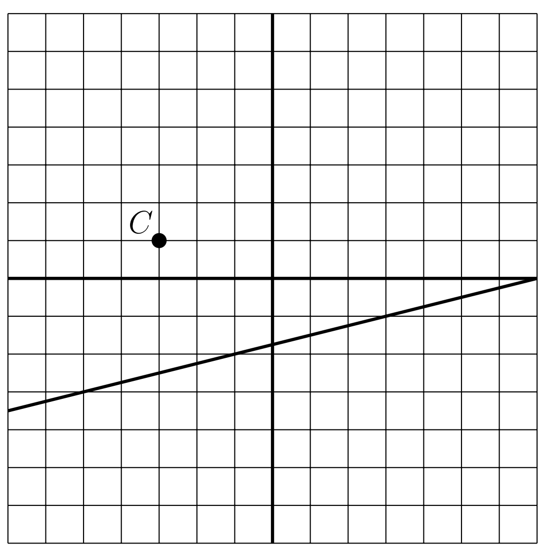


   (a) Hilda is recuperating from an accident and can't walk very far. She wants to know where she can go if she only walks two blocks. Draw the taxicab circle representing the places she can visit.

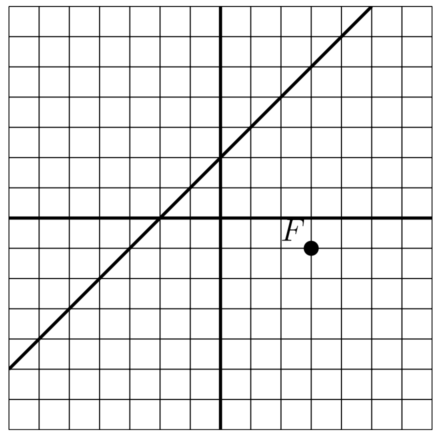
   (b) Hilda's recovery is proceeding well from week to week. Draw circles representing how far she can go if she walks 3 blocks, 4 blocks, or 5 blocks.

   (c) How far is Hilda's house from the bike path?

3. The bike path in Taxicab City runs on a line through $(-5,-3)$ and $(3,-1)$, as shown. How far is City Hall $C = (-3,1)$ from the bike path?



4. Fred is a city engineer preparing to hook up the electricity to Taxicab City's new football stadium at $F = (3, -1)$. He needs to hook into a main power line that runs along a line from $(-7, -5)$ to $(5, 7)$.

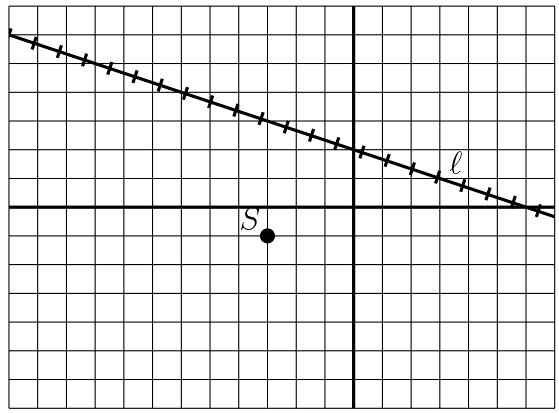


His cable needs to be buried along the city streets. How many blocks' worth of cable does he need to reach from the stadium to the power line? What route should the cable take?

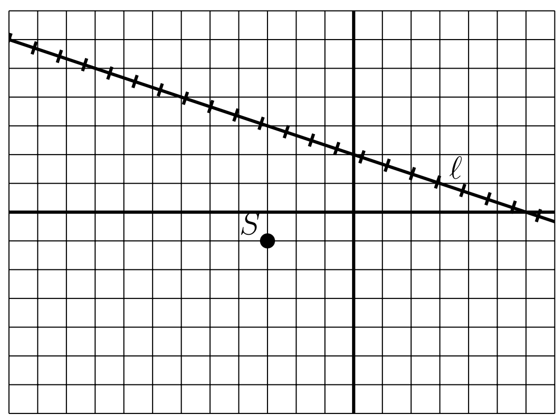
5. Formulate a rule for deciding how far a point is from a line in taxicab distance. (Hint: consider how steep the line is.)

6. Susan and Leo are moving to town. Susan got a job at Sushi Hut at $S = (-3, -1)$, whereas Leo will be working for the city light rail line ℓ that runs through the city as shown. One of Leo's fringe benefits is that when he comes to work he can just get on the train wherever is closest to his home.

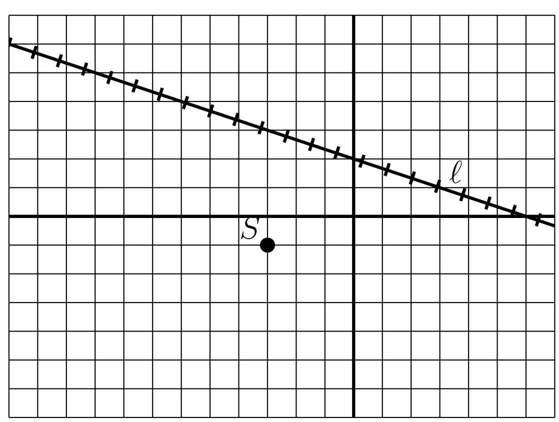
   (a) Susan and Leo want to live where the distance Susan has to walk to work plus the distance Leo has to walk to work is a minimum. Where should they look?



   (b) They change their minds and decide to live where they both walk the same distance to work. Where should they look?

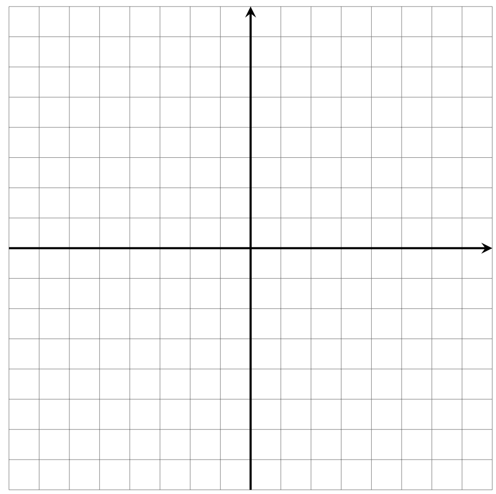


   (c) Where should they look if all that matters is that Susan have a shorter distance to walk than Leo?



Day 5: Taxicab Triangles

1. (a) Draw three right triangles on the graph.



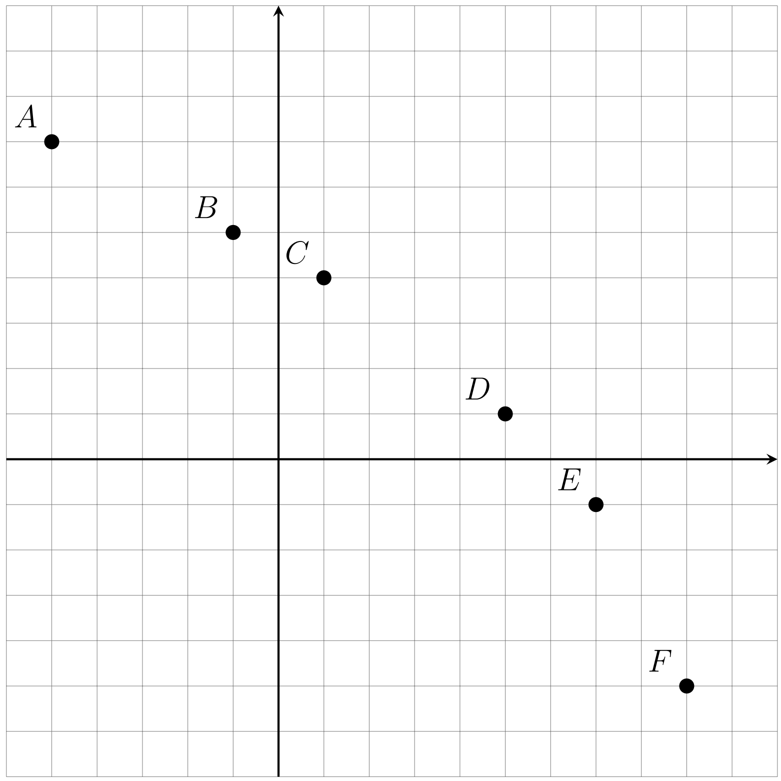
   (b) Measure their sides using taxicab distances.

	Leg (short side)	Leg (short side)	Hypotenuse (long side)
Triangle I			
Triangle II			
Triangle III			

   (c) Does the Pythagorean Theorem work in taxicab geometry? Why or why not?

2. Can you come up with a replacement for the Pythagorean Theorem in taxicab geometry? In other words, if you have a right triangle with legs of length a and b, can you find a formula for the length of the hypotenuse c?

3. Consider the points in the following diagram.



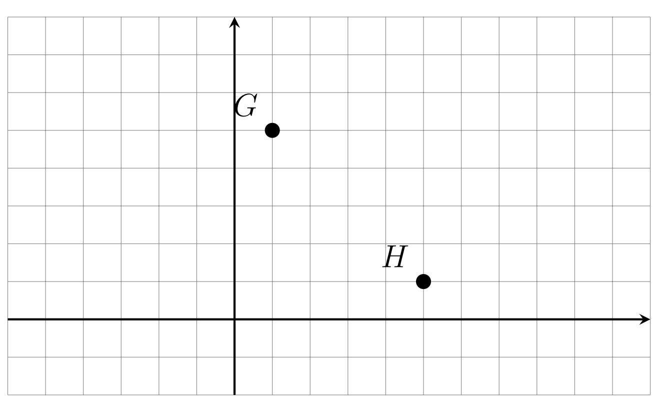
(a) Find all points that are the same distance from A as from B.

(b) Find all points that are the same distance from C as from D.

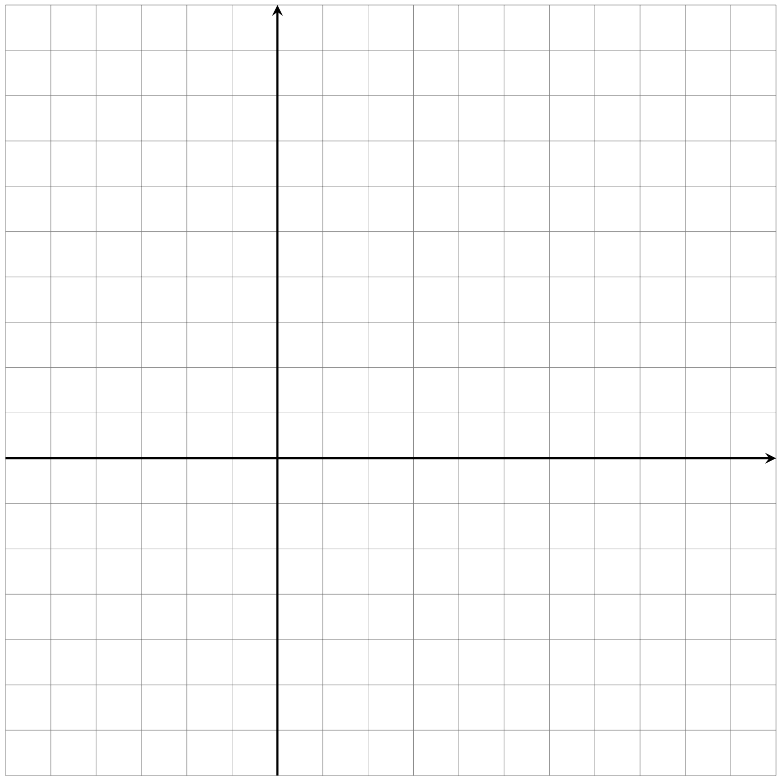
(c) Find all points that are the same distance from E as from F.

4. Given two points X and Y, describe a rule for finding all the points that are the same distance from X as from Y.

5. Find all points that are the same distance from G as from H.

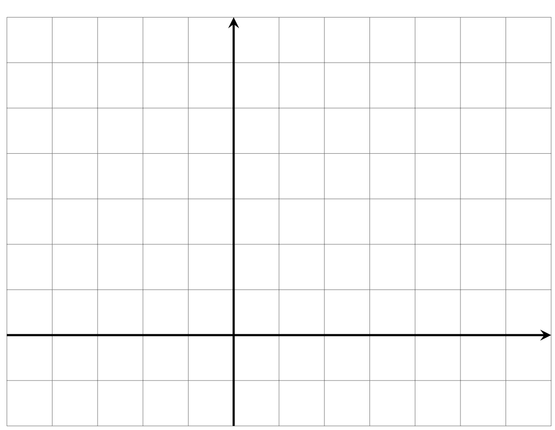


6. Let $A = (6, -2)$, $B = (0, 6)$, and $C = (8, 4)$.

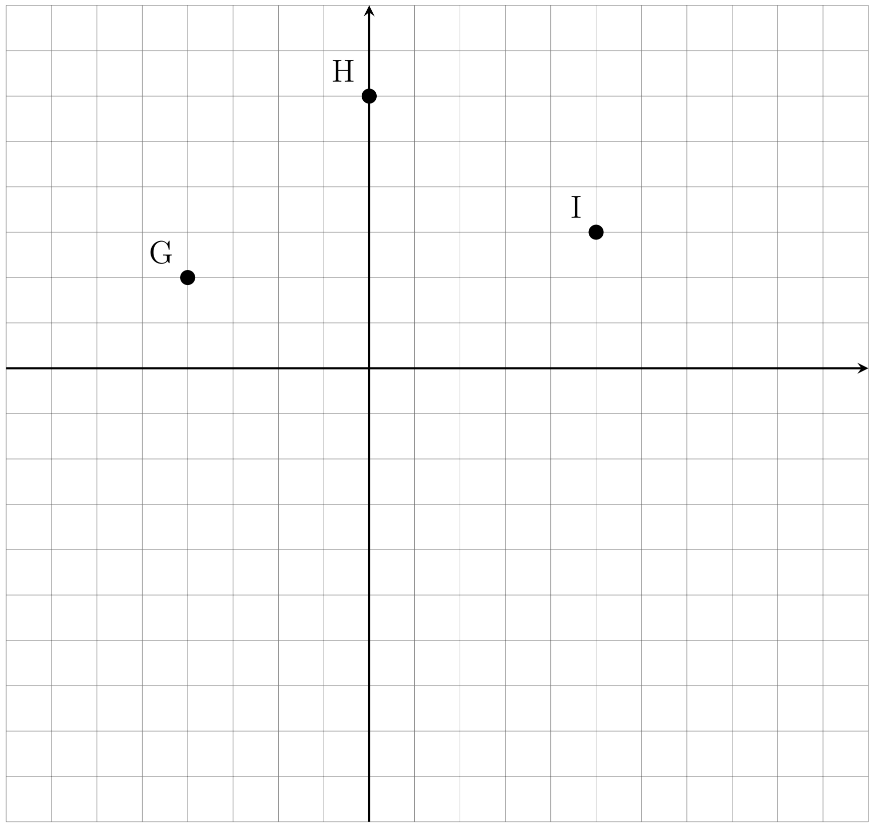


   (a) Draw the triangle $\triangle ABC$.
   (b) Find all points that are the same distance from A as they are from B.
   (c) Find all points that are the same distance from A as they are from C.
   (d) Find all points that are the same distance from B as they are from C.
   (e) Let P be the point where those three sets meet. What are the coordinates of P? How far is P from each of the three points A, B, and C?
   (f) Draw a taxicab-circle that touches each of the three corners of $\triangle ABC$.

7. Circumscribe a taxicab-circle around triangle $\triangle DEF$, where $D = (-3, 0)$, $E = (0, 1)$, and $F = (5, 5)$.



8. Consider triangle $\triangle GHI$, where $G = (-4, 2)$, $H = (0, 6)$, and $I = (5, 3)$.

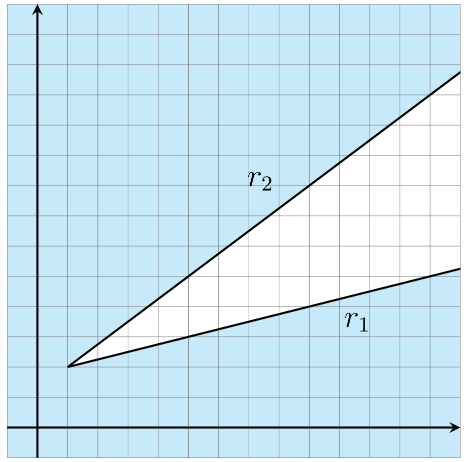


(a) Circumscribe a taxicab-circle around triangle $\triangle GHI$.

(b) Circumscribe a *different* taxicab-circle around $\triangle GHI$.

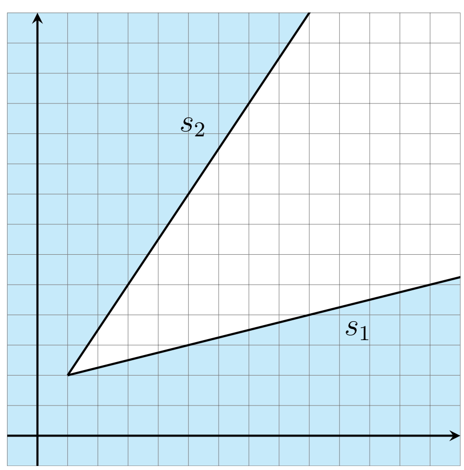
(c) How many different taxicab-circles *can* be circumscribed around $\triangle GHI$?

9. Consider the two rays (half-lines) r_1 and r_2 that make an angle in the following diagram.



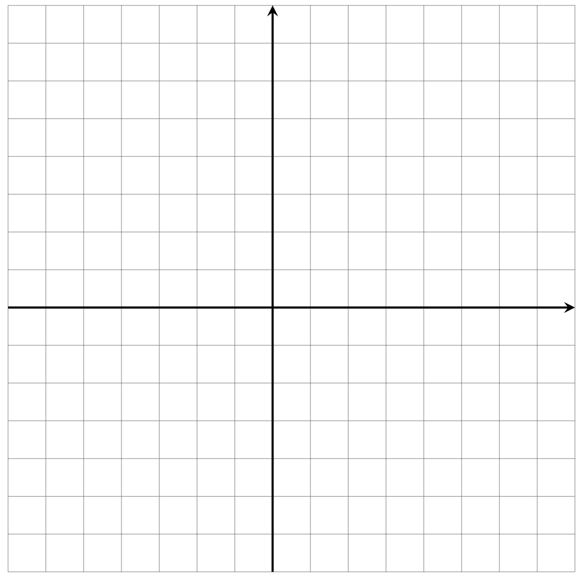
   (a) Is r_1 steep or shallow? Is r_2 steep or shallow?
   (b) Find a point A within the angle (unshaded region) such that the distance from A to r_1 is the same as the distance from A to r_2.
   (c) Find two more such points.
   (d) Connect the dots to find *all* points P in the angle such that the distance from P to r_1 is the same as the distance from P to r_2.

10. Consider the two rays (half-lines) s_1 and s_2 that make an angle in the following diagram.



   (a) Is s_1 steep or shallow? Is s_2 steep or shallow?
   (b) Find a point B within the angle (unshaded region) such that the distance from B to s_1 is the same as the distance from B to s_2.
   (c) Find two more such points.
   (d) Connect the dots to find *all* points P in the angle such that the distance from P to s_1 is the same as the distance from P to s_2.

11. Let $A = (-2, -5)$, $B = (-2, 4)$, and $C = (4, 7)$.



   (a) Draw the triangle $\triangle ABC$, which has three sides: side AB from A to B, side BC from B to C, and side AC from A to C.
   (b) Draw the line containing all points equally distant from side AB and side BC.
   (c) Draw the line containing all points equally distant from side AB and side AC.
   (d) Draw the line containing all points equally distant from side BC and side AC.
   (e) Let P be the point where the three lines meet. What are the coordinates of P? How far is P from each of the three sides?
   (f) Draw a taxicab-circle that touches each of the three sides of $\triangle ABC$.

12. Do you think that, given any triangle at all, it is possible to inscribe a taxicab-circle into it? Explain why or why not.