The Problem

How many ways are there to pick a committee of three from persons A, B, C, D, and E?

A Simpler Problem

How many ways to line up three of those people *in order*?

The Problem

How many ways are there to pick a committee of three from persons A, B, C, D, and E?

A Simpler Problem

How many ways to line up three of those people *in order*?

The Problem

How many ways are there to pick a committee of three from persons A, B, C, D, and E?

A Simpler Problem

How many ways to line up three of those people *in order*?

We start by counting the ways to choose three elements:

$$\_\_ \cdot \_\_ \cdot \_\_$$

The Problem

How many ways are there to pick a committee of three from persons A, B, C, D, and E?

A Simpler Problem

How many ways to line up three of those people *in order*?
We start by counting the ways to choose three elements:

$$\underline{5} \cdot \underline{\quad} \cdot \underline{\quad}$$

The Problem

How many ways are there to pick a committee of three from persons A, B, C, D, and E?

A Simpler Problem

How many ways to line up three of those people *in order*?
We start by counting the ways to choose three elements:

$$\underline{5} \cdot \underline{4} \cdot \underline{\quad}$$

The Problem

How many ways are there to pick a committee of three from persons A, B, C, D, and E?

A Simpler Problem

How many ways to line up three of those people *in order*?
We start by counting the ways to choose three elements:

$$\underline{5} \cdot \underline{4} \cdot \underline{3}$$

The Problem

How many ways are there to pick a committee of three from persons A, B, C, D, and E?

A Simpler Problem

How many ways to line up three of those people *in order*?
We start by counting the ways to choose three elements:

$$\underline{5} \cdot \underline{4} \cdot \underline{3} = 60$$

Sixty committees?

ABC	BAC	CAB	DAB	EAB
ABD	BAD	CAD	DAC	EAC
ABE	BAE	CAE	DAE	EAD
ACB	BCA	CBA	DBA	EBA
ACD	BCD	CBD	DBC	EBC
ACE	BCE	CBE	DBE	EBD
ADB	BDA	CDA	DCA	ECA
ADC	BDC	CDB	DCB	ECB
ADE	BDE	CDE	DCE	ECD
AEB	BEA	CEA	DEA	EDA
AEC	BEC	CEB	DEB	EDB
AED	BED	CED	DEC	EDC

Sixty committees?

ABC	BAC	CAB	DAB	EAB
ABD	BAD	CAD	DAC	EAC
ABE	BAE	CAE	DAE	EAD
ACB	BCA	CBA	DBA	EBA
ACD	BCD	CBD	DBC	EBC
ACE	BCE	CBE	DBE	EBD
ADB	BDA	CDA	DCA	ECA
ADC	BDC	CDB	DCB	ECB
ADE	BDE	CDE	DCE	ECD
AEB	BEA	CEA	DEA	EDA
AEC	BEC	CEB	DEB	EDB
AED	BED	CED	DEC	EDC

Sixty committees?

ABC	BAC	CAB	DAB	EAB	
ABD	BAD	CAD	DAC	EAC	
ABE	BAE	CAE	DAE	EAD	
ACB	BCA	CBA	DBA	EBA	No! These six are the same!
ACD	BCD	CBD	DBC	EBC	
ACE	BCE	CBE	DBE	EBD	
ADB	BDA	CDA	DCA	ECA	
ADC	BDC	CDB	DCB	ECB	
ADE	BDE	CDE	DCE	ECD	
AEB	BEA	CEA	DEA	EDA	
AEC	BEC	CEB	DEB	EDB	
AED	BED	CED	DEC	EDC	

Why 6?

- The single committee $\{A, B, C\}$ shows up once for each way it can be listed:
  ABC ACB BAC BCA CAB CBA
- How many ways can those three people be listed?

Why 6?

- The single committee $\{\mathrm{A}, \mathrm{B}, \mathrm{C}\}$ shows up once for each way it can be listed:
  ABC ACB BAC BCA CAB CBA

- How many ways can those three people be listed?

Why 6?

- The single committee $\{A, B, C\}$ shows up once for each way it can be listed:
  ABC ACB BAC BCA CAB CBA
- How many ways can those three people be listed?

Why 6?

- The single committee $\{\mathrm{A}, \mathrm{B}, \mathrm{C}\}$ shows up once for each way it can be listed:
  ABC ACB BAC BCA CAB CBA

- How many ways can those three people be listed?

  ___ ___ ___

Why 6?

- The single committee $\{A, B, C\}$ shows up once for each way it can be listed:
  ABC ACB BAC BCA CAB CBA

- How many ways can those three people be listed?

$$\underline{3}\quad \underline{\quad}\quad \underline{\quad}$$

Why 6?

- The single committee $\{A, B, C\}$ shows up once for each way it can be listed:
  ABC ACB BAC BCA CAB CBA

- How many ways can those three people be listed?

$$\underline{3} \cdot \underline{2} \quad \underline{}$$

Why 6?

- The single committee $\{A, B, C\}$ shows up once for each way it can be listed:
  ABC ACB BAC BCA CAB CBA
- How many ways can those three people be listed?

$$\underline{3} \cdot \underline{2} \cdot \underline{1}$$

Why 6?

- The single committee $\{A, B, C\}$ shows up once for each way it can be listed:
  ABC ACB BAC BCA CAB CBA

- How many ways can those three people be listed?

$$\underline{3} \cdot \underline{2} \cdot \underline{1} = 3! = 6$$

So how many?

ABC	BAC	CAB	DAB	EAB
ABD	BAD	CAD	DAC	EAC
ABE	BAE	CAE	DAE	EAD
ACB	BCA	CBA	DBA	EBA
ACD	BCD	CBD	DBC	EBC
ACE	BCE	CBE	DBE	EBD
ADB	BDA	CDA	DCA	ECA
ADC	BDC	CDB	DCB	ECB
ADE	BDE	CDE	DCE	ECD
AEB	BEA	CEA	DEA	EDA
AEC	BEC	CEB	DEB	EDB
AED	BED	CED	DEC	EDC

So how many?

ABC	BAC	CAB	DAB	EAB
ABD	BAD	CAD	DAC	EAC
ABE	BAE	CAE	DAE	EAD
ACB	BCA	CBA	DBA	EBA
ACD	BCD	CBD	DBC	EBC
ACE	BCE	CBE	DBE	EBD
ADB	BDA	CDA	DCA	ECA
ADC	BDC	CDB	DCB	ECB
ADE	BDE	CDE	DCE	ECD
AEB	BEA	CEA	DEA	EDA
AEC	BEC	CEB	DEB	EDB
AED	BED	CED	DEC	EDC

So how many?

ABC	BAC	CAB	DAB	EAB
ABD	BAD	CAD	DAC	EAC
ABE	BAE	CAE	DAE	EAD
ACB	BCA	CBA	DBA	EBA
ACD	BCD	CBD	DBC	EBC
ACE	BCE	CBE	DBE	EBD
ADB	BDA	CDA	DCA	ECA
ADC	BDC	CDB	DCB	ECB
ADE	BDE	CDE	DCE	ECD
AEB	BEA	CEA	DEA	EDA
AEC	BEC	CEB	DEB	EDB
AED	BED	CED	DEC	EDC

So how many?

ABC	BAC	CAB	DAB	EAB
ABD	BAD	CAD	DAC	EAC
ABE	BAE	CAE	DAE	EAD
ACB	BCA	CBA	DBA	EBA
ACD	BCD	CBD	DBC	EBC
ACE	BCE	CBE	DBE	EBD
ADB	BDA	CDA	DCA	ECA
ADC	BDC	CDB	DCB	ECB
ADE	BDE	CDE	DCE	ECD
AEB	BEA	CEA	DEA	EDA
AEC	BEC	CEB	DEB	EDB
AED	BED	CED	DEC	EDC

Ten!

ABC ACB BAC BCA CAB CBA	$\{A, B, C\}$
ABD ADB BAD BDA DAB DBA	$\{A, B, D\}$
ABE AEB BAE BEA EAB EBA	$\{A, B, E\}$
ACD ADC CAD CDA DAC DCA	$\{A, C, D\}$
ACE AEC CAE CEA EAC ECA	$\{A, C, E\}$
ADE AED DAE DEA EAD EDA	$\{A, D, E\}$
BCD BDC CBD CDB DBC DCB	$\{B, C, D\}$
BCE BEC CBE CEB EBC ECB	$\{B, C, E\}$
BDE BED DBE DEB EBD EDB	$\{B, D, E\}$
CDE CED DCE DEC ECD EDC	$\{C, D, E\}$

Ten!

ABC	ACB	BAC	BCA	CAB	CBA	$\{A, B, C\}$
ABD	ADB	BAD	BDA	DAB	DBA	$\{A, B, D\}$
ABE	AEB	BAE	BEA	EAB	EBA	$\{A, B, E\}$
ACD	ADC	CAD	CDA	DAC	DCA	$\{A, C, D\}$
ACE	AEC	CAE	CEA	EAC	ECA	$\{A, C, E\}$
ADE	AED	DAE	DEA	EAD	EDA	$\{A, D, E\}$
BCD	BDC	CBD	CDB	DBC	DCB	$\{B, C, D\}$
BCE	BEC	CBE	CEB	EBC	ECB	$\{B, C, E\}$
BDE	BED	DBE	DEB	EBD	EDB	$\{B, D, E\}$
CDE	CED	DCE	DEC	ECD	EDC	$\{C, D, E\}$

$$\frac{60}{3!} = 10$$

What Happened?

- $P(5,3) = 5 \cdot 4 \cdot 3 = 60$ lists of three people
- Each committee of three shows up in the list $P(3,3) = 3! = 6$ times
- Therefore the number of committees is

$$\frac{P(5,3)}{P(3,3)} = \frac{60}{6} = 10.$$

What Happened?

- $P(5,3) = 5 \cdot 4 \cdot 3 = 60$ lists of three people
- Each committee of three shows up in the list $P(3,3) = 3! = 6$ times
- Therefore the number of committees is

$$\frac{P(5,3)}{P(3,3)} = \frac{60}{6} = 10.$$

What Happened?

- $P(5,3) = 5 \cdot 4 \cdot 3 = 60$ lists of three people
- Each committee of three shows up in the list $P(3,3) = 3! = 6$ times
- Therefore the number of committees is

$$\frac{P(5,3)}{P(3,3)} = \frac{60}{6} = 10.$$

Conclusions

- In general, suppose you have n people and you want to choose r of them for a committee.
- Then there are $P(n, r)$ ways to choose them *in order*.
- Each committee shows up in that list $P(r, r) = r!$ times.
- Therefore the number of committees is

$$\frac{P(n, r)}{r!}$$

- This is called the ***number of combinations of*** n ***objects taken*** r ***at a time***.
- It is written as $C(n, r)$ and is pronounced "n choose r."

Conclusions

- In general, suppose you have n people and you want to choose r of them for a committee.
- Then there are $P(n, r)$ ways to choose them *in order*.
- Each committee shows up in that list $P(r, r) = r!$ times.
- Therefore the number of committees is

$$\frac{P(n, r)}{r!}$$

- This is called the ***number of combinations of*** n ***objects taken*** r ***at a time***.
- It is written as $C(n, r)$ and is pronounced "n choose r."

Conclusions

- In general, suppose you have n people and you want to choose r of them for a committee.
- Then there are $P(n, r)$ ways to choose them *in order*.
- Each committee shows up in that list $P(r, r) = r!$ times.
- Therefore the number of committees is

$$\frac{P(n, r)}{r!}$$

- This is called the ***number of combinations of*** n ***objects taken*** r ***at a time***.
- It is written as $C(n, r)$ and is pronounced "n choose r."

Conclusions

- In general, suppose you have n people and you want to choose r of them for a committee.
- Then there are $P(n, r)$ ways to choose them *in order*.
- Each committee shows up in that list $P(r, r) = r!$ times.
- Therefore the number of committees is

$$\frac{P(n, r)}{r!}$$

- This is called the ***number of combinations of*** n ***objects taken*** r ***at a time***.
- It is written as $C(n, r)$ and is pronounced "n choose r."

Conclusions

- In general, suppose you have n people and you want to choose r of them for a committee.
- Then there are $P(n, r)$ ways to choose them *in order*.
- Each committee shows up in that list $P(r, r) = r!$ times.
- Therefore the number of committees is

$$\frac{P(n, r)}{r!}$$

- This is called the ***number of combinations of*** n ***objects taken*** r ***at a time***.
- It is written as $C(n, r)$ and is pronounced "n choose r."

Conclusions

- In general, suppose you have n people and you want to choose r of them for a committee.
- Then there are $P(n,r)$ ways to choose them *in order*.
- Each committee shows up in that list $P(r,r) = r!$ times.
- Therefore the number of committees is

$$C(n,r) = \frac{P(n,r)}{r!}$$

- This is called the ***number of combinations of*** n ***objects taken*** r ***at a time***.
- It is written as $C(n,r)$ and is pronounced "n choose r."

Permutations vs. Combinations

Important Fact

The difference between permutations and combinations is the idea of ***order***.

Key Questions

When selecting objects, ask yourself:

- Would it count as a different result if I rearranged the objects I chose?
- Am I selecting these objects to play *different roles* in the problem?

If the answer to either question is yes, use Permutations; if the answer to both questions is no, use Combinations.

Permutations vs. Combinations

Important Fact

The difference between permutations and combinations is the idea of ***order***.

Key Questions

When selecting objects, ask yourself:

- Would it count as a different result if I rearranged the objects I chose?
- Am I selecting these objects to play *different roles* in the problem?

If the answer to either question is yes, use Permutations; if the answer to both questions is no, use Combinations.

Practice Problems

(cf. 5.5 #39–43)

1. Suppose that you have 35 songs on your MP3 player. How many ways can you make a playlist of 5 songs?
2. Of the 20 applicants for a job, 4 will be selected for intensive interviews. In how many ways can the selection be made?
3. In a batch of 100 USB drives, 7 are defective. A sample of three drives is to be selected from the batch. How many samples are possible? How many of the samples consist of all defective drives?
4. A student must choose five courses out of seven that he would like to take. How many possibilities are there?
5. How many different three-letter "words" are there having no repetition of letters?

A More Advanced Problem

6. A club has six people: Alice, Bob, Charlie, Danielle, Edgar, and Frederika. Alice and Danielle are twins, and so are Bob and Charlie. How many ways are there to line up the six people in a row for a picture, if we insist that each pair of twins is standing together?