contest with winners and losers. We need to work together to develop progressive goals for school mathematics and high-quality instructional resources. Most important, we need to change the tenor of public and professional discourse about mathematics education. We need to dial down the acrimonious policy arguments and relentless criticism of schools and teachers. We in the profession need to be articulate and persistent in making the case that teaching is one of the most important and demanding tasks for adults in our society and that teachers deserve our whole-hearted encouragement and support.

References


Demystifying the Math Myth: Analyzing the Contributing Factors for the Achievement Gap between Chinese and U.S. Students

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The fact that Chinese students outperform American students consistently in mathematics achievement has been widely recognized over the past 40 years (Ornstein, 2010). In this chapter we identify and analyze seven important factors that contribute to Chinese students’ superior math performance: culture, curriculum, textbooks, teachers, teaching, students, and parents. We dissect the influential factors where the differences lie, and describe the differences in math education between China and the United States. The work is based on an extensive literature review and systematic research employing multiple methods, including our distinctive firsthand learning, teaching, teacher-supervising, and research experiences both in China and in the United States; participation in the writing of the National Unified Textbook and Teachers Manual in China; and in both countries, multiple interviews of elementary students, parents, and math teachers; extensive observation of classroom teaching; and comparisons of math textbooks. In this work, “Chinese students” refer to students in mainland China.

Although it stands in unison with a previous work regarding cultural, societal, and familial differences between China and the United States (Chiu, 2014), this article distinguishes itself with its emphasis on an in-depth analysis of China’s national math curriculum and math textbooks, the use of specialized math teachers, and teacher support. Additionally and importantly, this chapter provides recommendations for the United States to improve students’ math achievement.
Factors Contributing to Mathematics Gap Between Chinese and American Students

Culture

Education Is Highly Prized in the Chinese Culture. Children’s education is the top priority in Chinese families. In modern China, education is believed to be the basis for social development as well as for individual prosperity. Children are instilled with the common knowledge that those with a higher academic background will have a greater chance for a higher-level life. Education has been one of the government’s central focuses in China. In the early 1970s, the Chinese government specified universal primary education as a national goal. In 1985, the government issued a reform plan that called for improving the educational level of the entire population by providing all children at least nine years of formal schooling.

An Intense Exam and Competition Culture Exists in China. Education competition and examination are prevalent in China and are highly regarded. China’s one set of national standards makes national comparisons possible. Chinese students are consistently in competitions with each other, and they depend on the National College Entrance Examination to move up the social ladder. China has an estimated 230 million K-12 students—roughly four times the combined U.S. public and private school population—and only a fraction of them get to enter college, based on higher academic performance. Students must fight for limited spaces in schools and universities. On the contrary, the United States is a wealthy nation, where students have myriad opportunities for educational and economic advancement.

Emphasis on Effort. The Chinese attribute success to effort, and failure to lack of enough effort; while Americans often tend to attribute success or failure to high or low ability, respectively. Attributing success to personal ability could prompt a person to give up when faced with failure. In contrast, attributing success to effort, which is within one’s control, often leads a person to sustain hope and persistence, and to increase effort.

Curriculum

A national curriculum contributes greatly to Asian students’ superior math performance (e.g., Schmidt, Houang, & Cogan, 2002; Stevenson & Stigler, 1992). There is a national unified mathematics curriculum used throughout all of China to guide textbook content, teacher training, and professional development. Four types of teaching materials—the Teaching and Learning Framework (教学大纲, jiao xue da gang), textbooks (课本, ke ben), teacher’s manuals (教师教学用书, jiaoshi jiaoxue yongshu), and workbooks (练习册, han xi ce)—constitute China’s national curriculum. A national curriculum provides a platform for professional exchanges and the establishment of a knowledge base for effective teaching (Hiebert, Gallimore, & Stigler, 2002).

The education system in the United States has been traditionally decentralized and the U.S. national government does not impose much control over education policy. The U.S. Department of Education is not a central ministry for education that oversees all related national and local services in education (Crow & Silver, 2008). The current American K-12 system has more than 13,000 school district governing bodies that hold almost complete control over the design, administration, and outcome of K-12 schools in the United States. These school boards are commonly led by elected groups of residents. Attaining national educational objectives through a network of independent local school boards is almost impossible.

U.S. mathematics curriculum materials are less focused and more repetitive in terms of content coverage, instructional requirements, and structures (Schmidt, Houang, & Wolfe, 1999), and curriculum policy is less authoritative, specific, and consistent (Wang, 2001). American schools end up circling back through topics over a student’s course of study, without teaching basic concepts to mastery (Cavanagh, 2006).

Textbooks

One vs. Many Series. There is one series of elementary math textbooks that has been used dominantly throughout China since 2001: The Compulsory Standardized Experimental Mathematics Series (义务教育课程标准实验数学教材) published by the People’s Education
Press (the PEP series). Besides the PEP series, there are five other series that are recognized and used, none of which has a comparable influence to the PEP series. In contrast, the decentralization of the U.S. education system has resulted in a much larger variety of textbooks being used in schools across the country. According to Usiskin and Dossey (2004), most school districts in the United States designed their own curriculum/standards within the guidance provided by individual states. The United States has never used a unified set of textbooks. In this study, we selected a representative mathematics textbook series developed by one of the most popular publishers and will refer it as “the U.S. series” for anonymity purposes, to compare with the Chinese PEP series.

Small and Light vs. Big and Heavy. The Chinese PEP series textbooks are consumable paperbacks that measure 5.83 inches in width × 8.27 inches in length, which is roughly half the size of the U.S. textbook. Each book weighs approximately 5 ounces. There are around 100 pages in each textbook. The U.S. series textbook is hardback, and measures approximately 9.2 inches in width by 11.2 inches in length. Each U.S. series textbook contains around 400 pages. They are colorful, and full of color photographs and colored illustrations and figures on almost every page. Each U.S. textbook weighs 2 pounds, 6 ounces, to over 3 pounds, nearly as much as all 12 PEP series textbooks put together. Table 1 provides a comparison of a Chinese PEP series math elementary textbook and the U.S. series typical elementary textbook. The cost of a Chinese math textbook is only a very small fraction of that of a U.S. math textbook.

Sequential and Intensive vs. Repetitive and Cursorry. China’s mathematics curriculum is very well sequentially organized, with almost no repetition. The textbooks are short and definitive. Different topics are taught in different grades. The same concept or skill becomes a subconcept or subskill for the advanced concept when it is presented at an advanced grade level. The books pursue a topic thoroughly within a particular grade and do not reteach it.

The U.S. mathematics textbooks reflect a spiraling curriculum, characterized by a great deal of repetition and review, with the result that topics are covered with little intensity (Kim, 1993). Unfortunately, a spiraling curriculum allows just a short time for the learning of each topic and frequently insufficient time for mastery of the topic. Succeeding years grant even less time to cover the material that was supposedly mastered in the previous year (Jiang & Eggleton, 1995). Many teachers feel strongly that they have insufficient time to go over the advanced levels or other topics. “There’s just too much stuff in it,” they say.

Chinese math textbooks cover more advanced topics and introduce topics at a faster rate than those in the United States. According to the report of the Illinois Council of Teachers of Mathematics (ICTM) China Mathematics Delegation (1988), “the Chinese students are learning a great deal of mathematics, . . . at any grade level, the students we observed seem ahead of our own in terms of both topics studied and mastery thereof. Their curriculum is more intensive” (p. 436). This difference continues to exist in the twenty-first century.

**Teachers**

**Extensive Content Knowledge.** Researchers reported that Chinese math teachers have a deeper understanding about mathematics and its representations (Ma, 1999). They are able to provide clearer explanations, use teaching time more efficiently, develop smoother pedagogical flow, and engage students in inquiry using whole-class instruction (Perry, 2000; Stigler & Hiebert, 1999). Additionally, Ma (1999) described a longitudinal coherence in Chinese teachers’ thinking about mathematics.

**Specialized Math Teachers.** Math teachers in China only teach math and typically only have one class preparation and can concentrate on mathematics and become very good at teaching it. Chinese math teachers teach less than three hours a day, while American teachers teach five hours a day on average. In the United States, elementary school teachers usually teach all the subjects. “We simply don’t have
the time to sit down to prepare lessons carefully," said a preservice math teacher in the United States. One of the most serious problems reported by over 50% of American teachers was the difficulty of meeting the demands on their time, which is five times higher than the percentage of Chinese teachers who reported such a problem (Stevenson & Stigler, 1992).

**Complete Teacher Support.** Math teachers in China have the national curriculum as a coherent guideline. They are also provided with all of the needed tools—teacher's manual, textbooks, workbooks, and other materials such as manipulatives. The teacher's manual serves as a rich resource and support to Chinese math teachers. In the manual, a detailed lesson plan is presented for each topic.

American math teachers are only given "a long list of ideas about what should be taught (known as standards) and market-driven textbooks that include something for everyone but very little guidance, tools, or training" (Schmidt, Houang, & Cogan, 2002, p. 10). Without a coherent national curriculum, the teachers in the United States are "simply doing what we asked them" to do: "Teach everything you can. Don't worry about depth. Your goal is to teach 35 things briefly, not 10 things well" (Schmidt, Houang, and Cogan, 2002, p. 13).

Chinese math teachers frequently have only 2 periods of instruction (out of 6) each day (with one class preparation) with 40 minutes per period, and thus they teach only 10–12 periods per week. They spend the rest of their time preparing lessons, grading homework, helping students, and developing high-order problem-solving exercises. In the United States, teachers have very heavy classroom teaching assignments every day, and typically only have one class period to prepare for classes. They commonly have three to four different class preparations per school day (Becker & Selter, 1996).

**Professional Development.** Chinese schools do far more to help teachers improve their classroom skills. An elementary school principal in Beijing was surprised to learn that many teachers in the United States teach 20 classes a week. Teachers at his school led only 10 or 12 classes, using the extra time to prepare lessons, meet in groups, and discuss classroom strategies. In China, each school has a small and a large teaching research group. Small teaching research group members are teachers teaching the same subject in the same grade. They often meet to discuss ways of teaching certain topics. The large teaching research group includes all teachers teaching the same subject in the entire school. They usually meet once a week for a half day to study and explore better ways to teach the subject in general.

China has a teaching research office at each level—town, district, city, province, and nation. Open sample lessons are provided regularly about once a month. After the open lessons, new research findings are disseminated by the teaching researchers at the meetings in a written form along with oral explanations. Mathematics journals have articles on teaching specific topics or lessons. High-quality sample lesson plans are provided by the People's Education Press for teachers' reference. One series of textbooks being used nationwide makes all these possible. In the United States, because of diverse textbooks, such research activities become unrealistic and impossible, even if the teachers had the time.

**Teaching**

**Adherence to Curriculum and Textbooks.** In China, the teaching of mathematics is guided by a national curriculum and a series of textbooks used virtually throughout the nation. The teachers strictly adhere to the curriculum, follow the teacher's manual, and study the teaching materials intensively (Ma, 1999). They know the exact content they should be teaching each day. To prepare math lessons, Chinese math teachers scrutinize textbooks and teacher's manuals for their content and analysis of concepts, rationale for presentation, suggested instructional methods, and sample problems (Zhou & Peverly, 2005).

American teachers know that there are misalignments between the textbooks and the state standards. They have to deviate from the textbook's content or sequence in order to meet the state standards. Further, most U.S. teachers hold the view that "good teachers do not follow textbooks, but instead make their own curriculum" (Ball & Cohen, 1996, p. 6). They tend to organize their classrooms much more according to their own desires, resulting in great variability among classrooms.

**Instructional Practices.** Chinese teachers tend to spend a considerably high percentage of time teaching certain knowledge or skills thoroughly using textbooks. They typically use whole-class instructional format in a formal and controlled manner in which all students
are on task and the teacher orchestrates all activities. The U.S. public school system does not provide strong administrative or professional instructional guidance; thus, teachers often determine their own instructional approaches. They could simply “close the door” and teach using any method or content they chose (Desimone et al., 2005).

Chinese teachers use manipulatives as part of the instruction to help students understand the concept or relationship. After using a manipulative, Chinese teachers spend a great deal of time analyzing what the manipulative shows. They always help students understand certain knowledge beginning at a concrete level to pictorial, then to abstract. In contrast, many U.S. teachers use manipulatives as the entire instruction. According to Cai (2004), Chinese math teachers discourage solution strategies that lack generality. In contrast, U.S. teachers heavily emphasize the pragmatic nature of mathematics: As long as it works, students can choose whatever representations and strategies they like.

Despite the general belief that Chinese education is traditional and teacher-centered, the whole-class time is not necessarily teacher-centered lecture. Chinese math teachers often use the whole-class time to engage students in discussions of math problems; therefore, it could be characterized as student-centered (Huang & Leung, 2004). In a Chinese math classroom, the students produce most of the mathematical statements and explanations, whereas teachers in the United States produce more mathematical explanations and statements than the students do (Schleppenback et al., 2007).

Learning Time. Prior to 1995, Chinese students attended school 240 days per year, 6 days a week (Monday through Saturday) for 40 weeks a year. Since 1995, students attend school for 5 days a week, for a total of 200 days a year. There are two long vacations separating the school year: the summer vacation and winter vacation. A school day in China starts early and ends later than in the United States. Students usually arrive at 7:30 for a self-study session in order to get ready for the four class periods in the morning. After the fourth lesson, usually around noon, students get a 1.5- to 2-hour lunch and nap break. Afternoon lessons usually run from 1:30 to 4:30 p.m. Each class period lasts about 40–45 minutes. Mathematics is often taught during the first two periods of the day because early morning is the time when students’ brains are believed to be the freshest and sharpest.

American students attend school 180 days per year. Mathematics lessons can be scheduled at any time of the school day. U.S. students often spent about twice as much time on language arts as they do on math. The reason, according to a graduate student in the United States, is “because it’s easier for teachers to teach reading. Some of them don’t know math well.”

Classroom: Serious Learning Place vs. Social Center. The classroom environment is quite different in the two countries. Given the fact that there are about as many elementary and high school students in China as there are people in the United States, one will not be surprised that the size of a Chinese class is usually more than twice as large as a class in the United States. However, one will be very surprised to see that classroom discipline problems are nearly nonexistent in a Chinese classroom. Students’ minds follow the teacher very closely.

In contrast, based on our extensive observations of classroom teaching sessions, students in a U.S. classroom seem to have much freedom to do many things other than learning (although sometimes permissions are needed), such as getting up, walking across the room to use a pencil sharpener, washing their hands, going to the school office, the library, or the bathroom. Teaching is interrupted constantly, which makes systematic teaching very difficult. While Chinese teachers spend little time on classroom management, U.S. teachers usually spend a great deal of time to keep class in order.

Homework. Homework is an important form of out-of-school learning. Chinese children are typically assigned homework on weekday evenings, weekends, as well as holiday breaks. American children usually receive homework on weekday evenings, not weekends or holiday breaks, and the amount of homework on weekday evenings is less than that of Chinese children. Chinese children spend much more time each day doing homework than American children.

Students

Great Sense of Responsibility. Being raised in a collectivist society, Chinese children work hard to bring honor to the family. Chinese children also assume the responsibility of taking care of their parents when they become elderly. Thus, Chinese children study hard to acquire a good living with financial stability.
Effort for Success. While U.S. students usually attribute mathematics performance to intelligence, their Chinese counterparts hardly ever attach importance to the intelligence difference; instead, they place greater emphasis on the role of effort (e.g., Stevenson et al., 1990). Students in China are deeply influenced by the traditional belief that "effort is everything." To them, perseverance spells success.

High Standards. Chinese students are more critical toward themselves. While 87% of American fifth-graders thought their parents were happy or very happy with their math performance, only 38% of Chinese fifth-graders thought their parents were happy or very happy. Similarly, 83% of American children thought their teacher was happy or very happy with their math performance, compared to 37% of the Chinese children (Stevenson, 1987).

Parents

Dedicated to Children's Education. Chinese parents live their lives centered around enabling their children to obtain the highest academic achievement, and to grow up into successful people (健康成功). Chinese parents see education as the avenue toward a better future. For peasant families, children's education is the way to improve the family's financial state in the future. Chinese families dedicate a large portion of their time and themselves to their children's schoolwork. Basic math skills are often taught at home even prior to formal schooling. Parents not only tutor their children at home but also help them regularly review math textbooks the students learned before.

Even though American parents also help their children to develop and be successful, they tend to show less commitment to and put less emphasis on their children's academic achievement. It's less typical for American parents to see academic activity as something their children should be involved in after school, and they do not spend a great amount of time monitoring and encouraging their child on academic work.

High Expectations. Like their children, Chinese parents believe that academic achievement is more a product of effort than of natural ability (Tusi, 2007) and the children should do well if they put in enough effort. Consequently, they are much more critical when it comes to their children's academic performance. In contrast, American parents tend to emphasize ability as the most important requisite for success, and therefore are less critical when faced with their children's low academic performance. Stevenson et al. (1990) reported that the average math score with which the American parents would be satisfied was in the 70s, while the average score with which Chinese parents would be satisfied was around the 90s.

Conclusions

Many factors may have contributed to the existing mathematics achievement gap between American and Chinese elementary students. Some of the leading factors appear to be culture, curriculum, textbooks, teachers, teaching, students, and parents. Among these, we especially stress the importance and the potential game-changing effect of a coherent, precisely defined national curriculum, a series of well-developed textbooks, and specialized math teachers at all grade levels. America needs a more consistent approach to teaching math to replace the potpourri of approaches used in states and school districts. It is apparent that improving American elementary student math performance depends on changes and adjustments at the policy and practice levels.

We also wish to emphasize that American math teachers need more solid math content knowledge. According to a famous Chinese saying, if you want to give the students one cup of water, you should have one bucket of water of your own. Ma's study (1999) calls for teachers to have profound understanding of fundamental mathematics. The American Council on Education (ACE) proclaimed that "a thorough grounding in college-level subject matter and professional competence in professional practice are necessary for good teaching. . . students learn more mathematics when their teachers report having taken more mathematics" (American Council on Education, 1999, p. 6).

We do not suggest that the United States should simply import China's model of curriculum and instructional practices. Teaching practices are culturally dependent and there are many challenges to transplanting effective teaching practices from one country to another.

It also should be noted that the American education system has many strengths of its own. To illustrate, the Chinese government is seeking to infuse more American-style flexibility into its math and science curriculum by placing less emphasis on exams and more focus on cultivating students' creative and analytical skills.

The uphill battle on the math gap will be strenuous. The encouraging news is that we have the hardworking and enthusiastic American teachers
on board with the reform mission. Teachers at all levels are aware of the mathematics reform discussion (Drake, 2006), and have indicated commitment to moving toward practice that reflects the tenets of reform. American math teachers need to be well trained in mathematics subject areas, be released from the unreasonable demand of teaching multiple subjects, and be equipped with a coherent curriculum, well-written textbooks, high-quality teacher’s manuals, and supplementary materials to get down to business in the classroom with high expectations for students in mathematical achievement. If we choose to follow such a path, we will have an opportunity to remain competitive globally.

Summary of Implications

Seven Important Factors Contributing to Chinese Students’ Better Mathematics Performance

- In Chinese culture education is highly prized, competition is intense, and effort is emphasized.
- China has a coherent, national unified mathematics curriculum used throughout the country to guide textbook content, teacher training, and professional development.
- China has one series of well-developed math textbooks that are sequential and intensive, used dominantly throughout China.
- The teachers are well prepared, specialized in extensive content knowledge, have clear guidance, complete teacher support, and effective professional development.
- The lessons are usually well prepared, with more math teaching guided by a national curriculum.
- The students have a great sense of responsibility, effort, and high standards.
- The parents are dedicated to children’s education and have high expectations for children.

Recommendations for the U.S. Mathematics Community

- Establish a coherent, clearly defined national curriculum.
- Develop a series of well-developed textbooks.
- Have more specialized and well-trained math teachers, with solid content knowledge at all grade levels.
- Make changes and adjustments at both the policy and practice levels to improve student math performance.

Recommendations for China’s Mathematics Community

- Infuse American-style flexibility into the math curriculum by placing less emphasis on exams and more focus on cultivating students’ creative and analytical skills.

References


The Pigeonhole Principle
Two Centuries before Dirichlet

BENOÎT RITTAUD AND ALBRECHT HEEFFER

The Pigeonhole Principle

The pigeonhole principle states that when objects are placed into boxes and when there are more objects than boxes, then there will be at least one box containing at least two objects. The most striking use of such a simple principle is probably Dirichlet’s theorem on Diophantine approximation: for any real number \( \theta \) and any positive integer \( Q \), there exist integers \( p \) and \( q \) with \( 0 < q \leq Q \) such that

\[
\left| \theta - \frac{p}{q} \right| < \frac{1}{qQ}. \tag{1}
\]

The name of Dirichlet is commonly associated with the pigeonhole principle, because it is widely believed that he was the first to state it. In the writings of Dirichlet, the application of the principle is to be found in 1842 in [7] and [5] (both reproduced in [8], see pp. 579–80 and 633–38) and, later, in [6]. It seems that no one knows any precise previous reference, even if 1834 is frequently mentioned as the year of the discovery (for some details on later references, see [15]). In [7] and [5], the pigeonhole principle is used to prove complex and multidimensional versions of equation 1. In the second reference, of 1863 (hence published four years after Dirichlet’s death), it is used to provide a proof of the existence of infinitely many integers \( x \) and \( y \) such that \( x^2 - y^2 D < 1 + 2\sqrt{D} \) (for \( D \) integer and not a perfect square) which does not rely on continued fractions. In these publications Dirichlet does not assign any name to the principle, nor does he pretend that this principle is new. In a later work he calls it the “Schubfachprinzip.”