16.6 Parametrized Surfaces and Their Areas

- Motivation:

- Parametrized surfaces $\mathcal{S}$ given by $\mathbf{r}(u,v) = \Big(x(u,v),\ y(u,v),\ z(u,v)\Big)$:

1. Determine the surface $\mathbf{r}(u,v) = \Big(u\cos v,\ u\sin v,\ u\Big)$.

2. Parametrize the surface $x^2 + y^2 + z^2 = 25$.

3. Identify the surface $\mathbf{r}(u,v) = \Big(R\cos u,\ R\sin u,\ v\Big)$.

- Tangent Plane: For $\mathbf{r}(u,v) = \Big(x(u,v),\ y(u,v),\ z(u,v)\Big)$, form the tangent vectors $\mathbf{r}_u$ and $\mathbf{r}_v$.

4. Find the tangent plane to the parametric surface $\mathbf{r}(u,v) = \left(u,\ 2v^2,\ u^2+v\right)$ at the point $(2,2,3)$.

- Surface Area:

- Surface Integral:

5. Find the surface area of the portion of the cone $\mathbf{r}(u,v) = \big(u,\ u\cos v,\ u\sin v\big)$ for $u \in [0,2]$, $v \in [0, 2\pi]$.

6. Find the surface area of the hemisphere $x^2 + y^2 + z^2 = R^2$ for $z \geq 0$.

7. Evaluate the surface integral $\iint_{\mathcal{S}} xz \, d\mathcal{S}$, where $\mathcal{S}$ is the part of the plane $x + y + z = 1$ in the first octant.

8. Set up but do not evaluate the iterated integral representing $\iint_{\mathcal{S}} xyz \, d\mathcal{S}$, where $\mathcal{S}$ is the portion of the surface $y^2 = x$ between the planes $z = 0$, $z = 4$, $y = 1$, $y = 2$.