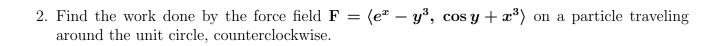
16.4 Green's Theorem

- Circulation—a line integral of a vector field around a closed curve. What is the circulation of a gradient vector field?
- Suppose \mathcal{C} is a simple closed curve that forms the boundary of a region \mathcal{D} in the xy-plane. Let \mathcal{C} be oriented so that traversing \mathcal{C} in the positive direction keeps \mathcal{D} to the left, i.e. counterclockwise orientation. The corresponding line integral of $\mathbf{F}(x,y) = \langle P(x,y), Q(x,y) \rangle$ around \mathcal{C} is denoted by $\oint_{\mathcal{C}} P(x,y) \ dx + Q(x,y) \ dy$.
- Green's Theorem: Let \mathcal{D} be a domain whose boundary $\mathcal{C} = \partial \mathcal{D}$ is a simple closed piecewise smooth curve in the plane, positively oriented (counterclockwise). If $\mathbf{F} = \langle P, Q \rangle$, where P(x, y) and Q(x, y) are continuous and have continuous partial derivatives, then

1. Evaluate $\oint_{\mathcal{C}} x^2 y \ dx + x \ dy$ using Green's Theorem, and then check the answer by evaluating the line integral directly, where \mathcal{C} is the triangle connecting the points (0,0), (1,0), and (1,2), oriented counterclockwise.



4. Use a line integral to find the area of the region enclosed by the astroid

$$x = a\cos^3 t$$
, $y = a\sin^3 t$, $0 \le t \le 2\pi$.

5. Use Green's Theorem to evaluate $\oint_{\mathcal{C}} \ln(1+y) dx - \frac{xy}{1+y} dy$, where \mathcal{C} is the triangle with vertices (0,0), (2,0), and (0,4), positively oriented.

6. Find the line integral of $\mathbf{F} = \langle x^2 + y, 4x - \cos y \rangle$ around the boundary of the region \mathcal{D} that is inside the square with vertices (0,0), (5,0), (5,5), and (0,5), but is outside the rectangle with vertices (1,1), (3,1), (3,2), and (1,2). Assume the boundary of \mathcal{D} is oriented such that \mathcal{D} is on the left when the boundary is traversed.