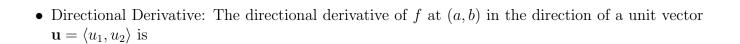
- Gradient Vector: If f is a function of two variables x and y, then the gradient of f is the vector function ∇f defined by
- 1. Find the gradient of $f(x,y) = \sin x + e^{xy}$ at (0,1).

2. The air temperature at ground level (x, y) is given (in Celsius) by the function

$$T(x,y) = 20 - x - y + 0.05x^3 - 0.2y^2.$$

Find the temperature gradient at the origin and at the point (-4,1). (See picture.)

• Recall for z = f(x, y) we defined the partial derivatives at (a, b) by



if this limit exists. If $\mathbf{u} = \mathbf{i} = \langle 1, 0 \rangle$, then $D_{\mathbf{i}} f(a, b) = f_x(a, b)$, and if $\mathbf{u} = \mathbf{j} = \langle 0, 1 \rangle$, then $D_{\mathbf{j}} f(a, b) = f_y(a, b)$.

• Theorem: If f is a differentiable function of x and y, then f has a directional derivative in the direction of any unit vector $\mathbf{u} = \langle u_1, u_2 \rangle$ and

Using the gradient vector we may rewrite the directional derivative in the direction of the unit vector \mathbf{u} as

3. Find the directional derivative of $f(x,y) = x^2y^3 - 4y$ at the point (2,-1) in the direction of the vector $\mathbf{v} = 2\mathbf{i} + 5\mathbf{j}$.

4. Find the directional derivative of $f(x,y) = xe^y + \cos(xy)$ at the point (2,0) in the direction of $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$.

5. Find the directional derivative of $f(x,y) = x^3 - 3xy + 4y^2$ in the direction $\theta = \pi/6$, that is, in the direction that points upward and to the right at an angle of θ . What is $D_{\bf u}f(1,2)$?

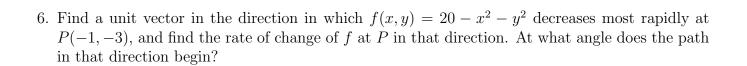
•	Three	Variables:	The	${\rm gradient}$	for	f(x, y, z) is	

and the directional derivative in the direction of a unit vector \mathbf{u} is

• Recall the angle between two vectors.

- \bullet Theorem: Suppose that f is a differentiable function of two (or three) variables. Then
 - (a) the maximum value of the directional derivative $D_{\mathbf{u}}f(x,y)$ (the maximum rate of change of f) at (a,b) is $|\nabla f(a,b)|$ and it occurs when \mathbf{u} has the same direction as the gradient vector $\nabla f(x,y)$: f increases most rapidly in the direction of the gradient;
 - (b) the minimum value of the directional derivative $D_{\mathbf{u}}f(x,y)$ (the minimum rate of change of f) at (a,b) is $-|\nabla f(a,b)|$ and it occurs when \mathbf{u} has the opposite direction as the gradient vector $\nabla f(x,y)$: f decreases most rapidly in the direction opposite the gradient;
 - (c) the rate of change of f at (a, b) is 0 in the directions orthogonal to $\nabla f(a, b)$;
 - (d) the gradient $\nabla f(a, b)$ is orthogonal to the level curve f(x, y) = c at the point (a, b), where c = f(a, b).

Proof:



7. If $f(x,y) = xe^y$, find the rate of change of f at the point P(2,0) in the direction from P to Q(1/2,2). In what direction does f have the maximum rate of change? What is this maximum rate of change?

8. Find the direction in which the function $f(x,y) = e^{2x+3y-1}$ increases most rapidly, starting at the point (3,7), and calculate its directional derivative in that direction.

9. If the elevation of a hill is given by $f(x,y) = 200 - y^2 - 4x^2$, in which direction will rain water run off the hill at the site (1,2)?