14.5 Chain Rule

1. $\boxed{\text{Chain Rule I}}$: Suppose that $z = f(x, y)$ is a differentiable function of x and y, where $x = g(t)$ and $y = h(t)$ are both differentiable functions of t. Then z is a differentiable function of t and

   Proof:

2. If $z = x^2y + xy^3$, where $x = \cos t$, $y = \sin t$, find dz/dt when $t = \pi/2$.

3. Find dz/dt if $z = \sqrt{x^2 + y^2}$ and $x = e^{2t}$ and $y = e^{-2t}$.

4. Suppose the production of a firm is modeled by the Cobb-Douglas production function

$$P(K, L) = 20K^{1/4}L^{3/4},$$

where K measures capital (in millions of dollars) and L measures the labor force (in thousands of workers). Suppose that when $L = 2$ and $K = 6$, the labor force is decreasing at the rate of 20 workers per year and capital is growing at the rate of $\$400,000$ per year. Determine the rate of change of production.

5. $\boxed{\text{Chain Rule II}}$: Suppose that $z = f(x, y)$ is a differentiable function of x and y, where $x = g(s, t)$ and $y = h(s, t)$ both have first partial derivatives with respect to s and t. Then

6. Find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$ for $z = \ln(x^2 + y^2)$, where $x = e^s \cos t$ and $y = e^s \sin t$.

7. Find $\dfrac{\partial w}{\partial s}$ and $\dfrac{\partial w}{\partial t}$ for $w = xy + xz + yz$, where $x = st$, $y = e^{st}$, $z = s + t$.

8. If $w = x^2 + y^2 + z^2$ and

$$x = \rho \sin\phi \cos\theta, \quad y = \rho \sin\phi \sin\theta, \quad z = \rho \cos\phi$$

find $\partial w/\partial \rho$ and $\partial w/\partial \theta$.

9. $\boxed{\text{Implicit Differentiation}}$: Suppose $\boldsymbol{F(x, y, z) = 0}$ implicitly defines a function $\boldsymbol{z = f(x, y)}$, where $\boldsymbol{f}$ is differentiable. Then

$$\boxed{\phantom{\dfrac{\partial z}{\partial x}}}$$

Proof:

10. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if $\boldsymbol{x^4 y + y^4 z + z^4 x = 5xyz}$.

11. Find the tangent plane to the surface $\boldsymbol{ze^z = x^2 - y^2}$ at the point $\boldsymbol{(1, 1, 0)}$.