13.4 Motion in Three-Space

1. Speed: $v(t) = s'(t) = |\mathbf{r}'(t)|$
   Velocity: $\mathbf{v}(t) = \mathbf{r}'(t)$
   Acceleration: $\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t)$

2. Find the velocity and acceleration functions for the position function $\mathbf{r}(t) = \langle te^{-2t}, 2e^{-2t}, -3t^2 \rangle$.

3. Find the velocity and position functions if $\mathbf{a}(t) = \langle t, 0, -4 \rangle$, $\mathbf{v}(0) = \langle 12, -4, 0 \rangle$, $\mathbf{r}(0) = \langle 5, 0, 2 \rangle$.

4. Newton's Second Law of Motion: $\mathbf{F} = m\mathbf{a}$, where $\mathbf{F}$ is the net force vector acting on the object, m is the mass, and $\mathbf{a}$ is the acceleration vector.

5. Projectile Motion: A projectile is launched with an initial speed of 49 meters per second from ground level at an angle of $\pi/4$ to the horizontal. Assuming the only force acting on the object is gravity (9.8 meters per second per second), find the

   (a) maximum altitude,

   (b) horizontal range, and

   (c) speed at impact of the projectile.

6. Tangential and Normal Components of Acceleration: Imagine an object moving along a curve determined by $\mathbf{r}(t)$. Recall that the tangent (velocity) vector is $\mathbf{v}(t) = \mathbf{r}'(t)$, and the speed is $v(t) = |\mathbf{v}(t)|$. Then $\mathbf{v}(t) = v(t)\mathbf{T}(t)$, and the acceleration $\mathbf{a}(t)$ of the object is given by

$$\mathbf{a}(t) = v'(t)\mathbf{T}(t) + \kappa(t)v^2(t)\mathbf{N}(t).$$

This says that the acceleration is always in the osculating $\mathbf{T}(t)\mathbf{N}(t)-$plane. The tangential and normal components of acceleration are then the coefficients $a_{\mathbf{T}} = v'(t)$ and $a_{\mathbf{N}} = \kappa(t)v^2(t)$, respectively.

We may also write

$$\mathbf{a}(t) = \left(\frac{\mathbf{a}(t)\cdot\mathbf{v}(t)}{|\mathbf{v}(t)|}\right)\mathbf{T}(t) + \left(\frac{|\mathbf{a}(t)\times\mathbf{v}(t)|}{|\mathbf{v}(t)|}\right)\mathbf{N}(t).$$

Proof:

7. A particle moves through 3-space such that its position vector at time t is $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$. Find the

   (a) scalar tangential and normal components of acceleration at time t.

   (b) scalar tangential and normal components of acceleration at time $t = 1$.

   (c) vector tangential and normal components of acceleration at time $t = 1$.

   (d) curvature of the path at the point at time $t = 1$.