

### 13.3b Curvature

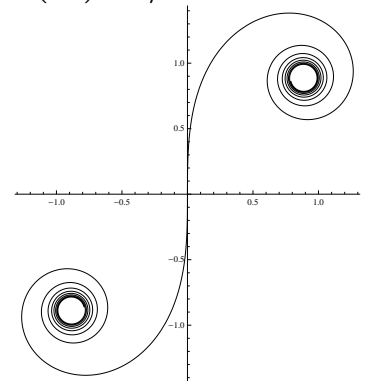
For practice: (860) 17,23,25,33,39,47,51a

ASSIGNMENT TO BE TURNED IN:

- Find the curvature of the helix  $\mathbf{r}(t) = \langle 2 \cos 2t, 2 \sin 2t, 3t \rangle$ .
- Calculate  $\mathbf{r}'(t)$  and  $\mathbf{T}(t)$ , and evaluate  $\mathbf{T}(1)$  for  $\mathbf{r}(t) = \langle 1 + 2t, t^2, 3 - t^2 \rangle$ .
- Find the curvature of the plane curve  $y = t^n$  at the point  $t = 1$ . Your answer will involve  $n$ .
- (a) Show that the curvature function of the parametrization  $\mathbf{r}(t) = \langle a \cos t, b \sin t \rangle$  of the ellipse  $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$  is

$$\kappa(t) = \frac{ab}{(b^2 \cos^2 t + a^2 \sin^2 t)^{3/2}}.$$

- (b) Use this equation for  $\kappa(t)$  to find the  $t$  values at which the maximum and minimum curvature occurs on the ellipse, assuming  $b > a$ . Lastly, what happens to the curvature if  $a = b$ ?
- The Cornu spiral is the plane curve  $\mathbf{r}(t) = \left\langle \int_0^t \sin\left(\frac{u^2}{2}\right) du, \int_0^t \cos\left(\frac{u^2}{2}\right) du \right\rangle$ . Find  $\kappa(t)$  for



the Cornu spiral; your answer should have an absolute value in it.

- Find the unit normal vector to the Cornu spiral (previous problem) at  $t = \sqrt{\pi}$ .