13.1 Vector-Valued Functions

1. Vector-Valued Function: A vector-valued function $\mathbf{r}(t)$ is a mapping from its domain $D \subset \mathbb{R}$ to its range $R \subset \mathbb{R}^3$ so that for each t in D we have $\mathbf{r}(t) = \mathbf{v}$ for exactly one vector $\mathbf{v} \in \mathbb{R}^3$. A vector-valued function can be written as

$$\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$$

using scalar functions f, g, h that are called component functions of $\mathbf{r}$.

2. Sketch the curve traced out by the endpoint of the two-dimensional vector-valued function

$$\mathbf{r}(t) = (t+1)\mathbf{i} + (t^2 - 2)\mathbf{j}.$$

3. Sketch the curve traced out by the endpoint of the vector-valued function

$$\mathbf{r}(t) = (4\cos t)\mathbf{i} - (3\sin t)\mathbf{j}, \quad t \in \mathbb{R}.$$

4. Sketch the curve traced out by the vector-valued function

$$\mathbf{r}(t) = (\sin t)\mathbf{i} - (3\cos t)\mathbf{j} + (2t)\mathbf{k}, \quad t \geq 0.$$

5. Match each of the vector-valued functions below with the corresponding computer-generated graph:

$$\mathbf{f}_1(t) = \langle \cos t, \ln t, \sin t \rangle \quad \mathbf{f}_2(t) = \langle t\cos t, t\sin t, t \rangle$$

$$\mathbf{f}_3(t) = \langle 3\sin 2t, t, t \rangle \qquad \mathbf{f}_4(t) = \langle 5\sin^3 t, 5\cos^3 t, t \rangle$$

6. Find parametric equations for the intersection of the cone $z = \sqrt{x^2 + y^2}$ and the plane $y + z = 2$.

7. Parametrize the intersection of the plane $y = \frac{1}{2}$ with the sphere $x^2 + y^2 + z^2 = 1$.

8. Describe the curve $\mathbf{r}(t) = (t\sin t)\mathbf{i} + (t\cos t)\mathbf{j} + t^2\mathbf{k}$ and its projections onto the $xy-$ and $yz-$planes. Find a circular paraboloid on which the curve lies.

9. Show that the graph $\mathbf{r}(t) = \langle 3\cos t, 3\sin t, 3\sin t\rangle$ lies on both a circular cylinder and a plane. What shape is it? Find the length of its major axis and its minor axis.

10. (847) #47. Two particles travel along space curves

$$\mathbf{r}_1(t) = \langle t^2, 7t - 12, t^2 \rangle \quad \text{and} \quad \mathbf{r}_2(t) = \langle 4t - 3, t^2, 5t - 6 \rangle,$$

respectively. Do the particles collide? Do the paths intersect?

11. Limits: For a vector-valued function $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$, the limit of $\mathbf{r}(t)$ as t approaches a is given by

$$\lim_{t \to a} \mathbf{r}(t) = \lim_{t \to a} \langle f(t), g(t), h(t) \rangle = \left\langle \lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t) \right\rangle,$$

provided all the limits exist. If any of the limits on the right-hand side above fail to exist, then $\lim_{t \to a} \mathbf{r}(t)$ does not exist.

12. Find the domain of $\left\langle \frac{3}{t^2}, \frac{\ln t}{t^2 - 1}, \sin 2t \right\rangle$, and $\lim_{t \to 1} \left\langle \frac{3}{t^2}, \frac{\ln t}{t^2 - 1}, \sin 2t \right\rangle$.

13. Continuity: The vector-valued function $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$ is continuous at $t = a$ whenever $\lim_{t \to a} \mathbf{r}(t) = \mathbf{r}(a)$.

14. Theorem on Continuity: The vector-valued function $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$ is continuous at $t = a$ if and only if the component functions f, g, h are all continuous at $t = a$.

15. Determine the values of t where the vector-valued function

$$\mathbf{r}(t) = \left\langle \frac{1}{t} \sin 2\pi t, \tan 2\pi t, \cos 2\pi t \right\rangle$$

is continuous.