Calculus III Double Homework for Friday

Section 12.5: (824) 5, 7, 11, 13, 29, 31, 45; 4, 10, 12, 14, 26, 30, 34

On a separate piece of paper, also do the following exercises.

E1. Find two unit vectors in $\mathbb{R}^2$ that make an angle of 45° with $4\mathbf{i} + 3\mathbf{j}$.

E2. Prove the following:

(a) For vectors $\mathbf{a}$ and $\mathbf{b}$ in $\mathbb{R}^3$,

$$|\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2 \, |\mathbf{b}|^2 - (\mathbf{a} \cdot \mathbf{b})^2.$$

(b) If $\mathbf{a}$ and $\mathbf{b}$ are nonzero vectors with $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a} \times \mathbf{b}|$, what is the angle between $\mathbf{a}$ and $\mathbf{b}$? Write your answer in degrees.

E3. Find a vector orthogonal to the plane through the points $P(1, 0, 0)$, $Q(0, 2, 0)$, $R(0, 0, 3)$ and find the area of the triangle PQR.

E4. Find the center and radius of the sphere $x^2 + y^2 + z^2 = 4x - 2y$.

E5. Find an equation of a sphere if one of its diameters has endpoints $(2, 1, 4)$ and $(4, 3, 10)$.

E6. On the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z = 3$, find the points closest to and farthest from the origin.

E7. Find equations of the spheres with center $(2, -3, 6)$ that touch

(a) the $xy-$plane;

(b) the $yz-$plane;

(c) the $xz-$plane.

E8. Describe in words the region of $\mathbb{R}^3$ represented by $1 \leq x^2 + y^2 + z^2 \leq 25$.

E9. Consider the points P such that the distance from P to $A(-1, 5, 3)$ is twice the distance from P to $B(6, 2, -2)$. What is the set of all such points P? Be as specific as possible.

E10. Find an equation of the set of all points equidistant from the points $P(6, 2, -2)$ and $Q(-1, 5, 3)$. Describe this set.