

12.2 Vectors in 2D and 3D

1. Vector and Magnitude:
2. Examples:
3. Equivalent Vectors:
4. Vector Addition (Parallelogram Law):
5. Scalar Multiplication:
6. Position Vectors:
7. Vector Equality:
8. Finding a position vector: Find the vector with
 - (a) initial point $A(-3, 2)$ and terminal point $B(1, 4)$;
 - (b) initial point $A(a_1, a_2)$ and terminal point $B(b_1, b_2)$.
 - (c) Find $|\overrightarrow{AB}|$.

9. Vector Addition/Subtraction:

10. Scalar Multiplication:

11. Parallel:

12. For vectors $\mathbf{v} = \langle 4, -2 \rangle$ and $\mathbf{w} = \langle -3, 5 \rangle$, compute

(a) $\mathbf{v} + \mathbf{w}$

(b) $\frac{1}{2}\mathbf{v} + 10\mathbf{w}$

(c) $5\mathbf{v} - 3\mathbf{w}$

(d) $|-2\mathbf{w}|$

13. Are $\mathbf{v} = \langle -4, 6 \rangle$ and $\mathbf{w} = \langle 3, -5 \rangle$ parallel?

Pick x so that \mathbf{v} and $\mathbf{u} = \langle 3, x \rangle$ are parallel.

14. Standard Basis Vectors \mathbf{i} , \mathbf{j} , \mathbf{k} :

15. Algebraic Properties: For any vectors \mathbf{u} , \mathbf{v} , and \mathbf{w} and real scalars c and d :

(a) $\mathbf{v} + \mathbf{w} = \mathbf{w} + \mathbf{v}$	Commutativity
(b) $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$	Associativity
(c) $\mathbf{v} + \mathbf{0} = \mathbf{v}$	Additive Identity
(d) $\mathbf{v} + (-\mathbf{v}) = \mathbf{v} - \mathbf{v} = \mathbf{0}$	Additive Inverse
(e) $c(\mathbf{v} + \mathbf{w}) = c\mathbf{v} + c\mathbf{w}$	Distributive law
(f) $(c + d)\mathbf{v} = c\mathbf{v} + d\mathbf{v}$	Distributive law
(g) $(1)\mathbf{v} = \mathbf{v}$	Multiplication by 1
(h) $(0)\mathbf{v} = \mathbf{0}$	Multiplication by 0

16. Unit vectors:

17. Find a unit vector in the same direction as $\mathbf{v} = \langle -2, 5, -9 \rangle$.

18. Find a vector with magnitude 6 in the direction of $\langle 2, 2, -1 \rangle$.

19. The thrust of an airplane's engine produces a speed of 966 kph in still air. The plane points in the direction of $\langle 2, 2, 1 \rangle$, and the wind is given by $\langle 16, -32, 0 \rangle$ kph. Find the velocity vector of the plane with respect to the ground and find the speed.

20. An airplane has an airspeed of 650 kilometers per hour (kph). Suppose the wind velocity is given by the vector $\mathbf{w} = \langle 32, 48 \rangle$. In what direction should the airplane head in order to fly due west, in other words in the direction of $-\mathbf{i}$?
21. Two forces \mathbf{F}_1 and \mathbf{F}_2 with magnitudes 10 lbs and 12 lbs, respectively, act on an object at a point P . Find the resultant force \mathbf{F} , its magnitude, and its angle. (See diagram)