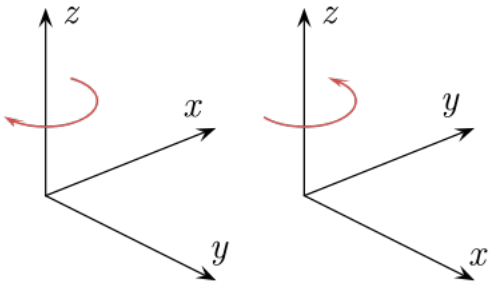
12.1 Three-Dimensional Coordinate Systems

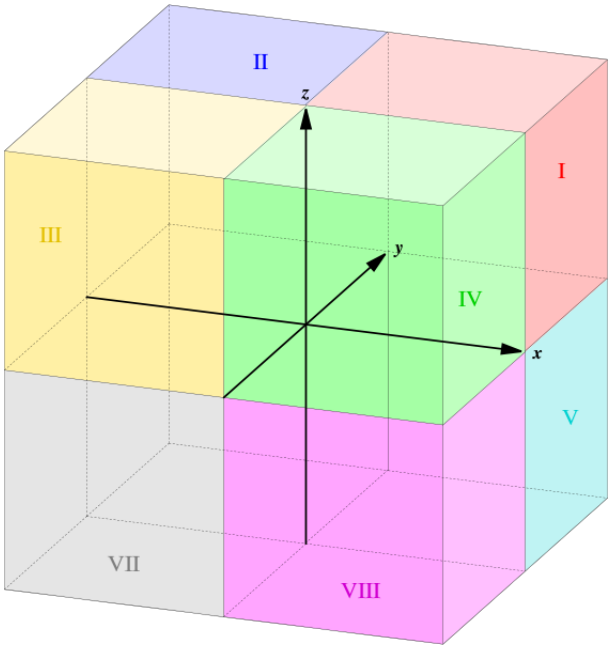
1. We consider points (x, y, z) in 3-dimensional Euclidean space $\mathbb{R}^3$, usually consisting of three rectangular coordinate axes x, y, z that intersect at the origin point $O = (0, 0, 0)$.

2. Handedness:



Left-handed on the left, Right-handed on the right

3. Octants:



4. Planes & Axes:

Region	Description
$xy-$plane	
$xz-$plane	
$yz-$plane	
$x-$axis	
$y-$axis	
$z-$axis	

5. Plotting (a, b, c) for $a, b, c > 0$: Move a units along the x-axis from the origin O to the point $(a, 0, 0)$, then parallel to the y-axis b units to the point $(a, b, 0)$, then c units parallel to the z-axis to the point (a, b, c). Move similarly for other coordinates.

6. Plot the points $(1, -2, -3)$ and $(-3, 1, 2)$.

7. Distance: The distance between $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is given by

   $$|P_1P_2| =$$

8. Sphere: A sphere is the set of all points at a constant distance (radius) from a given point (center). Therefore,

   describes sphere with points (x, y, z) centered at (a, b, c) and radius r.

9. Find the distance between $(-1, 0, 2)$ and $(1, 2, 3)$.

10. Identify the following:

    - $(x-3)^2 + (y-2)^2 + (z-1)^2 = 9$
    - $(x+1)^2 + y^2 + (z+4)^2 = 5$
    - $x^2 + y^2 + z^2 = 1$

11. Identify the geometric shape described by the equation

$$x^2 + 2x + y^2 + z^2 - 4z = 0.$$

12. Describe the set of all points in 3−space whose coordinates satisfy $x^2 + y^2 + z^2 - 2x + 8z \leq 8$.

13. Describe the set of all points in 3−space whose coordinates satisfy $y^2 + z^2 + 6y - 4z > 3$.

14. Sketch the complete graph of $x^2 + z^2 = 1$ in 3−space.

15. Are the points $P(2, 3, 1)$, $Q(4, 2, 2)$, and $R(8, 0, 4)$ on the same line?